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Statics-based Contact Behavior Simulation of a Manipulated Object

Yasushi Ikei, Ken Takahashi, and Shuichi Fukuda

Department of Production, Information, and Systems Engineering
Tokyo Metropolitan Institute of Technology
6-6 Asahigaoka, Hino, Tokyo 191-0065 JAPAN
ikei@tmit.ac.jp

Abstract

In this paper, we describe a behavior simulation method of a manipulated object based on statics. We assume a scene where an object is manipulated by the user's hand grasping the object at a point; the hand and the object are connected by a virtual spring. The condition simulates low speed manipulation of an object at near the surface with friction, against that the object collides. The contact between the object and surfaces is classified into three states. The algorithm to solve equilibrium equations for each state is presented, and the results of the simulation are shown by their trajectory data. The simulation system proved to enable a realistic stick-slip behavior of a manipulated object.

Key words: Contact behavior, Statics-based simulation, Surface friction, Object manipulation, Simplified physics

1. Introduction

For building a virtual space with an effective interaction to a spatial object, the object's behavior needs to be presented in an appropriate manner. Physics simulation with spatial objects plays a crucial part in virtual environment applications that are essentially related to physical aspects of objects. Such virtual space applications as training on apparatus manipulation or on surgical operations depend strongly on a realistic behavior of tool objects in their successful use. Moreover, the evaluation of a design product in terms of its manipulation properties is another instance that needs physical behavior simulation.

The simulation of object's behavior based on physical properties has been long discussed in terms of off-line computational dynamics [1,2]. Real-time simulation is a more recent topic which intends to construct an interactive virtual environment (VE). The simulation methods include those targeted to unconstrained motion of rigid bodies with friction [3,4], by impulse-based methods [5,6]. Those for constrained motion also have plenty of papers in robotics [7,8]. Another approach to VE simulation is to introduce arbitrary laws for manipulation convenience [9-11].

In this paper, we seek a method to calculate constrained motion by approximating physics with a focus to a major effect at the contact. The statics balance under existence of friction is only incorporated into the simulation in order to reduce the complexity of realtime calculation. The complete simulation of physical laws in the real world is almost impossible when the number of objects in VE increases due to application requirements. Even if the dynamics is omitted, the statics calculation can provide sufficiently realistic behavior of an object where it is manipulated at a relatively low speed.

2. Model environment with statics

Complexity of simulation is much alleviated by incorporating only statics, omitting the dynamics calculation of an object's behavior. Nevertheless, a certain reality can be achieved with only statics, if the motion is constrained by the user's hand and that the characteristics due to weight and/or inertial momentum of objects have little effect. The statics gives the relation between forces that balance, thereby the configuration (position/orientation) of objects can be determined provided that the boundary conditions are given by the user's input and other objets in the environment.

We consider a method to simulate the behavior of an object held and manipulated by the user, involving contacts to another object, and the resultant constraint motion during the manipulation. To investigate the fundamental component of such behavior calculation, we discuss here a two dimensional (2D) rectangle object model as illustrated in Figure 1. The object is essentially a two dimensional polygon, the position and the orientation of which are manipulated by the user's hand. The object can contact to the fixed surfaces of a floor and/or a wall at its vertices or at its edge. At these contact points, a Coulomb friction is assumed.

The user controls the object by a 3D input device (with six degrees of freedom), although the statics is calculated only for the three degrees of freedom, a 2D translation and a rotation. The user holds the object at a point---the initial holding point---from which the hand position goes off when the object collides and is constrained at opposite surfaces. The two positions, the initial holding position and the current hand position, are assumed to be connected by a virtual spring. Therefore, a force and a moment are exerted to the object when the two points have a distance in translations and orientations between them. This condition corresponds to where the user holds the object loosely with the fingers that bend compliantly against the resistive force from colliding points.

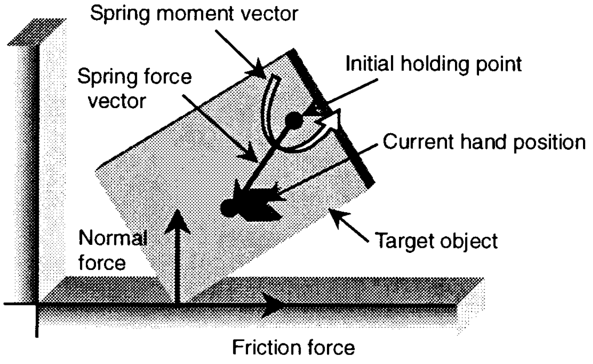

Figure 1 A model of a spatial object and a hand coupled by a virtual spring

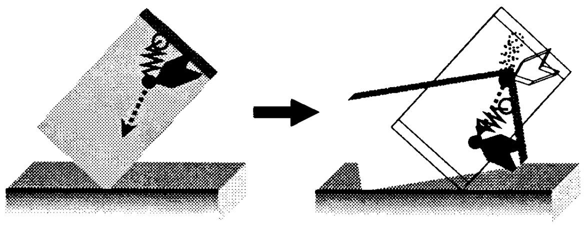

Figure 2 Stick-slip behavior with friction

This model enables to replicate a stick-slip behavior on a surface with friction in addition to a constraint motion or a rotation on the surface. (Figure 2) These behaviors are effectual for enhancing a realistic representation of an object during a positioning or an assembly task of a spatial object.

3. State transitions

Object behavior is calculated based on the contact states that occur during the manipulation. We consider here the four contacting states of the simulated object to other fixed surfaces. The four contact states are as follows.

a) Non-contact

b) Contacting at a single vertex to a surface

c) Contacting at an edge to a surface

d) Contacting at two vertices to two surfaces

The state transition diagram is illustrated in Figure 3. The transition occurs when a stable configuration (position/orientation) is not possible for the boundary conditions introduced at the moment by an updated user's hand position. If the boundary condition does not yield a stable solution with regard to a static balance equation, we search a stable solution with another boundary condition which would be given after a virtual slip motion within the same contact state. If, again, no configuration is stable, we select a state change.

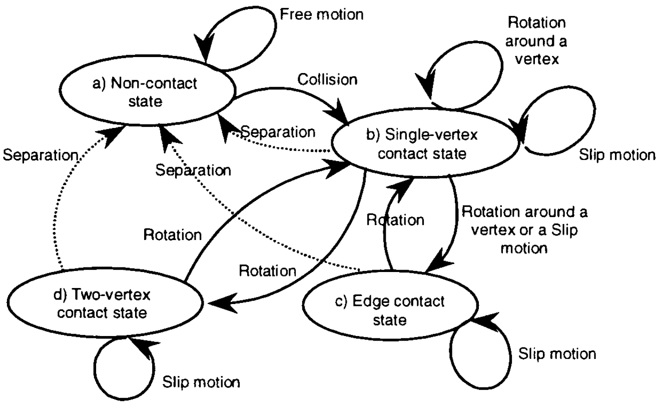

Figure 3 Contact state transition diagram

4. Statics balance equations

The equations of statics are described as follows. Equations (1) and (2) denote the balance of forces and moments exerted to the object, where $\boldsymbol{F}_h$ is the force of the user's hand; $\boldsymbol{F}_{ni}$ and $\boldsymbol{F}_{fi}$ are a surface normal force and a tangential friction force at the contacting vertex i. $\boldsymbol{M}_h$ is the moment added by the user's hand at the holding point. $\boldsymbol{M}_n$ and $\boldsymbol{M}_f$ are the moments caused by $\boldsymbol{F}_{ni}$ and $\boldsymbol{F}_{fi}$ at any point of interest, respectively.

$$\sum\left(F_h + F_{f_i} + F_{n_i}\right) = 0 \tag{1}$$

$$\sum\left(M_h + M_{f_i} + M_{n_i}\right) = 0 \tag{2}$$

The force and moment of the user's hand are assumed to be caused by virtual springs specified with Hooke's law shown in the equations

$$F_h = k_h \cdot \Delta\gamma\,, \tag{3}$$

$$M_h = k_\theta \cdot \Delta\theta\,, \tag{4}$$

where k_h, k_θ are the stiffness constants of the springs. $\Delta\gamma$ is a relative vector from the initial holding point on the object to the current hand position in the object's local coordinate. $\Delta\theta$ is a relative rotation angle of the hand from the initial holding orientation to the current orientation in the local coordinate.

The friction force has two forms, static or dynamic, according to the Coulomb model. The static friction is described as

$$\left|F_f\right| \le \mu_s \cdot F_n, \quad F_n > 0, \tag{5}$$

where μ_s is the coefficient of friction in a static condition. The direction and the magnitude of a static friction are determined by considering the contact state change incorporating the potential energy of a virtual spring and the natural constraint of directions of friction forces on a rigid body. The dynamic friction satisfies

$$\left|F_f\right| = \mu_d \cdot F_n, \quad F_n > 0, \tag{6}$$

where μ_d is the coefficient of friction with a sliding contact. The direction of F_f opposes that of the velocity of the contact point on a rigid body.

The equilibrium equations of (1) and (2) produce simultaneous equations of the forces and the object's orientation angle. The equations are non-linear since the hand's force and moment are functions of object's orientation angle to the contacting surface. To solve the equation, some iterative methods as the Newton's method or the secant method may be used. What should be noted is that the solution derived is not unique in most cases, which requires selection of an appropriate answer from them.

5. Solutions

The calculation to solve these equations differs among the three contact states. The first state following to the non-contact state is always a single-vertex contact state. The collision detection procedure determines one vertex of the object and the first contacting point on the colliding surface by interpolating the vertex's positions in the simulation frames before and after the collision. If a stable configuration was not found in the single-vertex contact state, the current state goes to one among the edge contact, the two-vertex contact and the non-contact state.

5.1. Single vertex contact

For the single vertex contact state (Figure 1), the flow of calculation is illustrated in Figure 4. After the determination of the colliding vertex and the position on the floor/wall surface with the first collision, the configuration---the position and orientation---of the object is calculated according to the equation of static equilibrium under the condition that the vertex is fixed on a surface to a point around which a rotation is yet allowed.

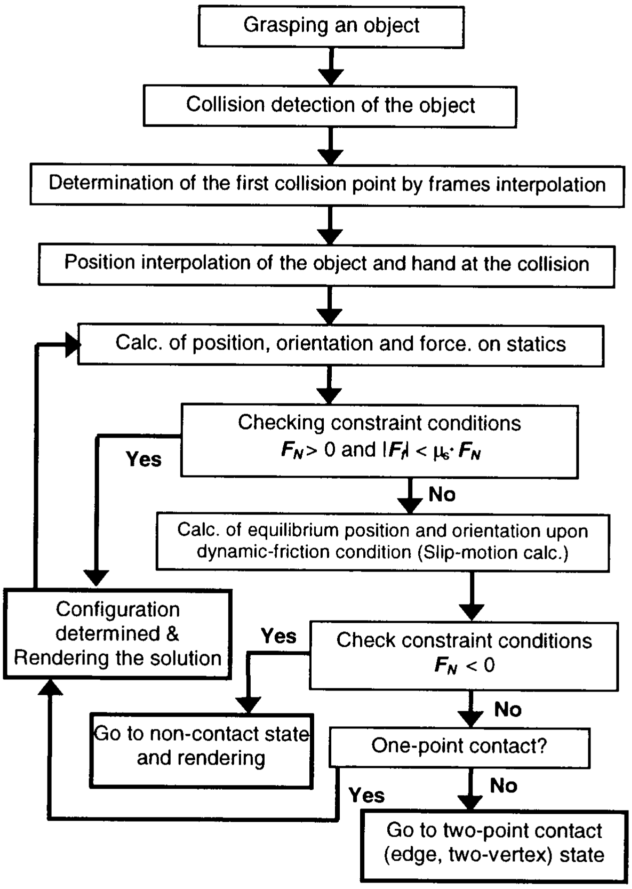

Figure 4 Calculation flow of single-vertex contact state

The equation, in this case, yields a unique solution of the object's orientation angle. In that solution, a static friction force and a surface normal force can take any large values or negative values. If they suffice the equation (5), the configuration is physically stable. Then, the calculation terminates, followed by rendering the configuration on the screen. Otherwise, the solution is not stable, where a slip motion within a single contact state is assumed to occur.

The slip motion itself is not calculated because it is a problem of dynamics. We calculate only the endpoint of the slip motion that occurs after the equilibrium under a dynamic friction (6) stood. Nominally, four solutions at most are obtained with that equation. Those are from the plus/minus sign (direction) of friction force, and from the two different states of kinetic energy of a virtual spring. We choose one solution which has a friction force directed opposing to the direction of hand's tangential force, and with the lower kinetic energy than before the slip.

We have to examine again the sign of the surface normal force; if it is negative, the configuration is not stable so that the current state goes to the non-contact state. We recalculate the position of the object according only to the hand position. If the surface normal force is positive, we examine if other vertices are colliding or not. This determines the current state to either the single contact or the two-vertex (edge) contact.

5.2. Edge contact

For the edge contact state, we approximate it as only the two vertices at the edge's end contact to the surface. (Figure 5) The solution is unique and resolved without an iterative method, if no slip motion is allowed. However, the friction forces at two vertices are not separately determined because the number of equations is less than that of the variables.

Figure 6 shows the calculation flow. First, the second contacting vertex is determined followed by the calculation of configuration at the collision. If the surface normal forces and the friction force suffice the constraint of the equation (5), the edge contact state is stable and the solution is rendered. Otherwise, the paths to resolve the state are complicated. If this unique solution does not meet a stable condition, the object will

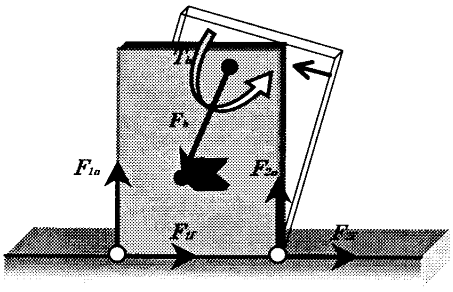

Figure 5 Edge contact state (calculated by two-vertex contact)

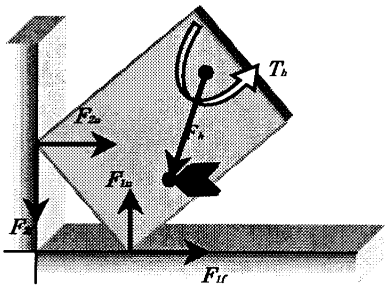

Figure 7 Two-vertex contact state

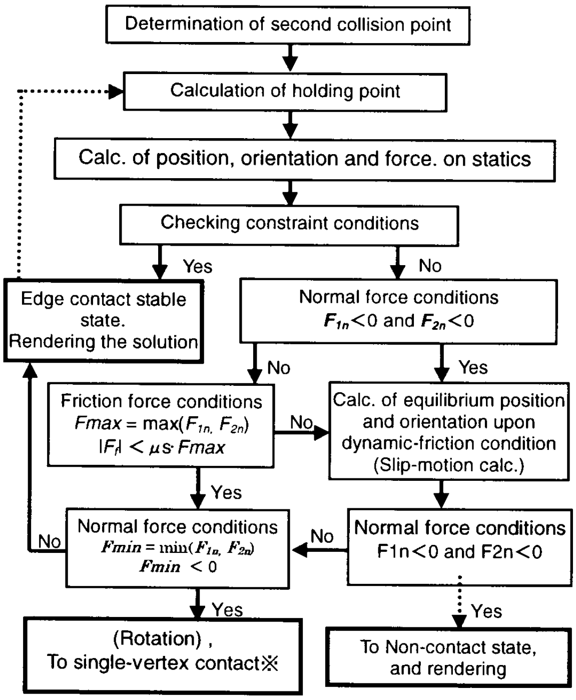

Figure 6 Calculation flow of edge contact state

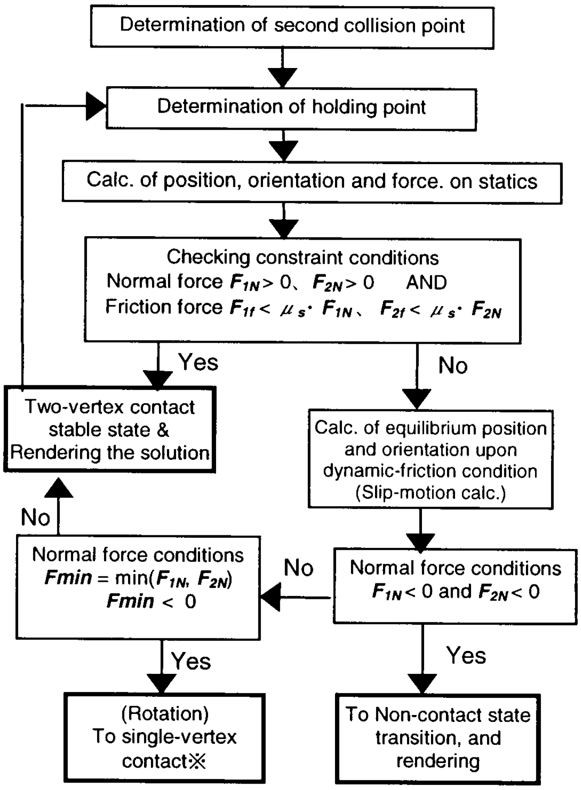

Figure 8 Calculation flow of two-vertex contact state

either slip or rotate on a vertex, involving the case of moving to the non-contact state. The slip end point is calculated under a dynamic friction condition, where the sign of friction force is set to oppose the direction of the user's hand tangential force.

5.3. Two-vertex contact

For the two-vertex contact state depicted in Figure 7, a unique solution does not exist under no slip condition, due to the lack of equations against variables. However, there are cases in which a stable solution exists. We search any valid solution that suffices constraints, because the difference between valid solutions that have different inner force distributions, makes no effect on the obtained configuration.

The calculation flow is shown in Figure 8. After the determination of the second collision point and the configuration at the moment, one stable solution is obtained through an iterative calculation. If the solution suffices the constraints, the configuration is rendered. Otherwise, a slip motion is assumed to give an end point that is determined under a dynamic friction condition. The end point calculation yields four solutions at most, within which we select a solution that holds the following conditions: the both frictions result in the same moment with respect to the initial holding point, and the potential energy of the virtual spring is smaller than before the slip. After the slip, if the surface normal forces are negative again, the current state goes to the non-contact. If one of the normal forces is negative, the object rotates at the other vertex and the current status goes to the single vertex contact.

5. Simulation results

The algorithm described above was implemented with World Tool Kit (Sense8 Inc.), the software tool to build a virtual space. An ultra-sonic three-dimensional tracker (Logitech Inc.) was used as an input device. The hold-release control of a virtual hand was done through a switch on the device. The object held by the user had four degrees of freedom (translation-x, y, z, and a rotation around an axis of depth), however the behavior calculation has three degrees of freedom. We can set four parameters, hand's translation stiffness k_h, hand's rotation stiffness k_θ, the coefficient of static friction μ_s, and the coefficient of dynamic friction μ_d to appropriate values. μ_d was set to 0.3 for the experiment. The frame rate was approximately 31 Hz on the Intergraph TD-30.

Figures 9 and 10 show the trajectories of the user's hand, the object, and forces under the single-vertex contact state. The trajectories show a trace of stick-slip motion that caused a jump in positions and equilibrium forces. Figures 11 and 12 depict those of the two-vertex contact state. An incontinuous stick-slip rotation with the contacts to the wall and the floor surfaces is also

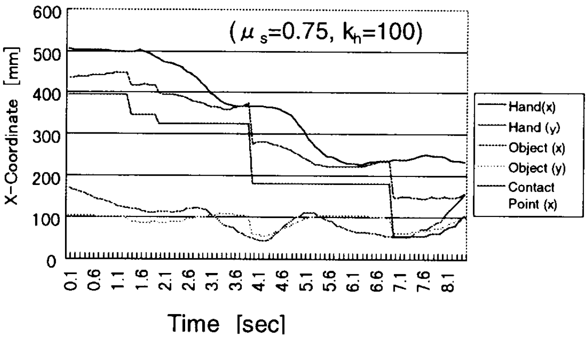

Figure 9 Trajectories of stick-slip motion under the single-vertex contact state

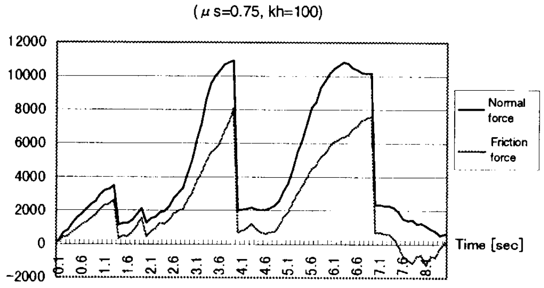

Figure 10 Normal and friction forces under the single-vertex contact state

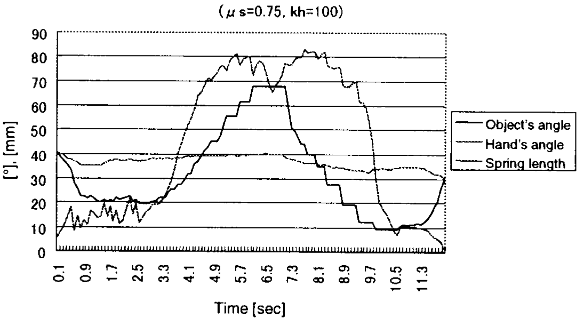

Figure 11 Trajectories of stick-slip motion under the two-vertex contact state

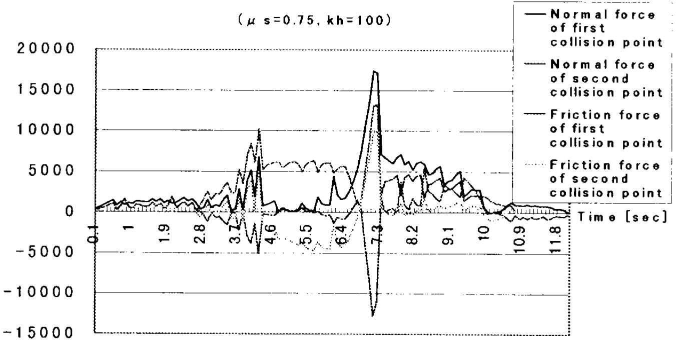

Figure 12 Normal and friction forces under the two-vertex contact state

observed.

Figures 13 and 14 are the snapshots of the simulation environment. In Figure 13, the object is about to slip to the left with the edge contact state. The magnitude of surface normal forces and a friction force is depicted by the length of a bar originated from the contact points. Figure 14 shows the counterclockwise rotation with a double contact to the floor and the wall at the vertices.

6. Conclusion and future work

We have developed a simulation system that enables a realistic behavior of manipulated object's motion with only the statics calculation. The contact phenomena between the object's vertices and fixed walls were calculated incorporating the Coulomb friction model. The object's state was classified into four contact conditions, between which the transitions were successfully switched to achieve a natural motion. The implemented system demonstrated the sufficient functionality of this fundamental element of simplified physics calculation.

The future work may include the extension of the algorithm to a six-degrees-of-freedom model for object description. In that way, simplification of the calculation model will be required still more to enable realtime simulation. Another interest is to examine the applicability of this calculation model to a force representation of stick-slip motion of a manipulated object. Not only the coefficient of friction but the surface texture property is dominant characteristics in the case of a slow speed contact phenomena. Therefore, both features should be discussed in terms of the contact behavior and force representation.

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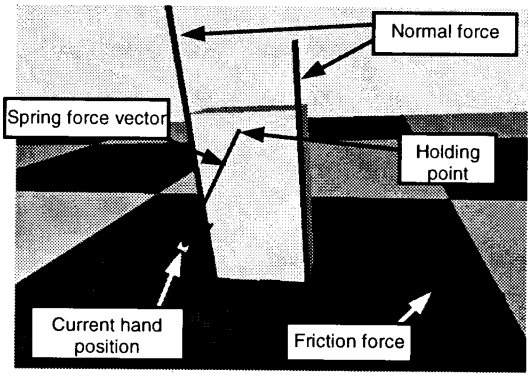

Figure 13 A scene of the edge contact state

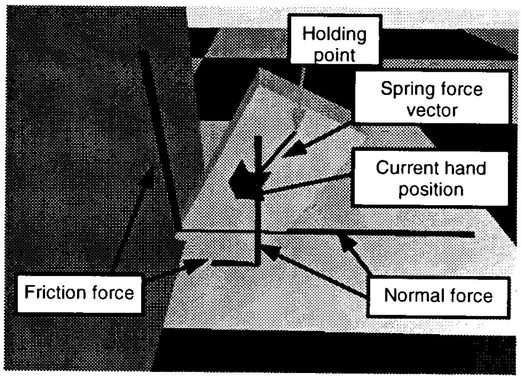

Figure 14 A scene of the two-vertex contact state