A Tactile Display Using Elastic Waves in a Tapered Plate

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Abstract

A tactile display using elastic waves in a 'tapered plate' (a plate whose width decreases gradually) is proposed. When an elastic wave is inputted to the tapered plate, there exists a boundary beyond which an amplitude of the wave is attenuated. The boundary is called 'turning point', and its position on the plate is controlled by the frequency of the wave. It is reported that touching a vibrating object with ultrasonic frequency generates a sense of a slippery surface. Therefore, by elastic wave in the tapered plate, a boundary between slippery and frictional area can be moved by the position of the turning point. Furthermore, because the amplitude of the wave is peaky at the turning point, surface textures are also controlled. In an experimental device employing a rubber plate, these basic phenomena are confirmed.

Key words: Tactile display, Elastic waves, Tapered plate

1 Introduction

Haptic sensation is divided into two parts. One is proprioception, which is sensation of weight, resistance, or the approximate shape of an object. The other is tactile, cutaneous sensation, which is a sense of roughness, ruggedness, or the otherwise variegated texture of an object's surface. The purpose of this study is to develop a tactile display that provides a tactile sensation in active touch.

We proposed a tactile display using elastic waves.[1] In our display, a spatial amplitude-modulated elastic wave is touched in our display. Finger skin detects the envelope of the A.M. wave whose wavelength is freely controlled. Theoretically, an arbitrary surface shape is generated as the envelope of the spatial A.M. wave. However, in the previously described display, only two sinusoidal waves are superposed, thus the envelope shape is confined to a simple sinusoidal shape.

In this paper, we propose a new method to create a more complicated texture on an elastic plate surface. We use elastic waves in a 'tapered plate'- a plate whose width decreases gradually and continuously. When an elastic wave is inputted into the tapered plate, an amplitude of the wave increases gradually from the place where the wave is inputted to a certain place called 'turning point'. After passing through the turning point, the amplitude of the wave is attenuated exponentially. The position of the turning point on the plate is controlled by the input frequency of the wave. It is reported that touching a vibrating object with ultrasonic frequency creates a sensation of a slippery surface by air lubrication called squeeze effect.[2] Therefore, an elastic wave in the tapered plate provides an slippery area from the place where the wave is inputted to the turning point , and an frictional area beyond the turning point. The boundary between the slippery and frictional areas can be moved by the wave frequency. By this method, the frictional state of the plate surface is controlled.

Furthermore, the amplitude distribution of the wave in the tapered plate is peaky at the turning point. Thus in a linear elastic body, waves with several frequencies creates several peaks whose positions are changed by the frequencies. Because this shape generates a spatial distribution of squeeze force, tactile sensation of surface textures can be controled.

An overview of this paper is as follows. In Section 2, we present an argument regarding elastic waves in a tapered plate. This phenomenon is usually not the focus of detailed argument in elastic waves theory[3] [4], but is fundamental to our display. Therefore, we analyze amplitude distribution of the elastic waves in the tapered plate at length. Based on elastic waves in an ordinary straight plate in 2.1, the basic equation and period equation of elastic waves in the tapered plate are derived from 2.2 to 2.5. We find analogy between the tapered plate equation and the Schrödinger equation in quantum mechanics, and introduce a concept of the potential energy of the tapered plate for elastic waves in 2.6. Using this concept of potential energy, detailed amplitude distribution is derived analytically in 2.7. In Section 3, we discuss the tactile display using elastic waves in a tapered plate. From the analysis in Section 2, the position of the turning point is represented by the wave frequency. By changing the frequency, the friction and the textures of the plate surface is controlled. In Section 4, an experimental device is constructed using a rubber plate as the elastic plate, and a oice coil as a vibrator. These basic phenomena are confirmed with the experimental display.

2 Elastic Waves in a Tapered Plate

2.1 Infinite Straight Plate

SH waves in an infinite elastic plate whose sides are fixed are considered here. We set the x axis along wave propagation direction, the z axis for plate width direction, and the y axis perpendicular to the other axes. A plate width is $2W_0$.

SH waves have an amplitude for the y direction. The following notation is used:

v : Displacement for y direction
μ : Rigidity of the elastic plate
ρ : Density of the plate

First, a displacement of the elastic plate for the y direction; v satisfies the wave equation;

$$\rho\frac{\partial^2 v}{\partial t^2} = \mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)v. \quad (1)$$

The solution for this equation is assumed as

$$v = jv_0\cos k_1 z\cdot \mathrm{e}^{-j(\omega t-kx)}. \quad (2)$$

Inserting (2) into (1), we obtain

$$\rho\omega^2-\mu(k^2+k_1^2) = 0. \quad (3)$$

From a condition of fixed sides, a wavenumber of the z axis; k_1 holds for

$$\cos k_1 W_0 = 0$$
$$\Longleftrightarrow k_1 W_0 = m\frac{\pi}{2}, \quad m=1,3,5... \quad (4)$$

Thus, inserting (4) into (3),

$$\left(\frac{\omega}{V_s}\right)^2 = \left(\frac{m\pi}{2W_0}\right)^2+k^2 \quad (5)$$

is obtained where V_s is S wave velocity of the elastic plate defined as

$$V_s = \sqrt{\frac{\mu}{\rho}}. \quad (6)$$

(5) is called a period equation. When a vibration with temporal frequency ω perpendicular to the plate is given, a traveling wave generated on the infinite elastic plate has a spatial frequency k for the x direction which satisfies (5). A dispersion curve(the realation between k and ω) of a rubber plate used in a later experiment is drawn in Fig. 2(a), where a plate has a width W_0 = 50[mm]. The right hand of the horizontal axis represents a real part of the wavenumber k, and the left hand of the horizontal axis represents an imaginary part of k. (In Fig.2(a), the first mode is drawn; m=1)

A node of the frequency curve and an axis of ordinates is determined from the wave width W_0 as

$$\omega_0 = \frac{\pi V_s}{2W_0}. \quad (7)$$

This is called cut-off frequency under which the wavenumber becomes imaginary and the wave is attenuated.

2.2 Tapered Plate

Now, a plate with a variable width according to $z = W(x)$ is considered.(Fig.1) We call a plate whose width decreases monotonously and gradually a 'tapered plate'. Elastic waves in a tapered plate are considered from this section.

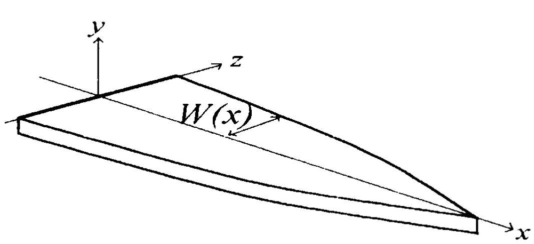

Figure 1: Tapered plate

In the microvolume of a tapered plate, an amplitude v should satisfy a wave equation as in a straight plate.

Boundary conditions in the tapered plate problem are different from those in the straight plate problem.

Wave equation:

$$\frac{1}{V_s^2}\frac{\partial^2 v}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)v. \quad (8)$$

Boundary conditions:

$$v=0 \qquad \text{at} \quad z=\pm W(x). \quad (9)$$

To solve a tapered plate problem, the transformation of variables is conducted as follows.

2.3 Basis of a tapered plate probrem

Now, curvilinear coordinates (α,β) defined as follows are introduced:

$$\alpha = x \quad (10)$$

$$\beta = \frac{z}{W(x)} \quad (11)$$

In (α,β) cordinate, boundary conditions become

$$v(\beta)=0 \qquad \text{at} \quad \beta=\pm 1, \quad (12)$$

which is treated as a straight plate problem with a constant plate width.

Lemma If $W\prime(x) << 1$, that is, if a change rate of the plate width is small enough, (α,β) is rectangular coordinates.

$$\begin{aligned}\text{proof)}\quad \nabla\alpha\cdot\nabla\beta &= \left(\frac{\partial\alpha}{\partial x},\frac{\partial\alpha}{\partial z}\right)\cdot\left(\frac{\partial\beta}{\partial x},\frac{\partial\beta}{\partial z}\right)\\ &= (1,0)\cdot\left(-z\frac{W\prime}{W^2},\frac{1}{W}\right)\\ &= -z\frac{W\prime}{W^2}\\ &= 0 \quad \text{Q.E.D}\end{aligned}$$

At this time, a stroke and the Laplacian are

$$\begin{aligned} ds^2 &= dx^2+dz^2\\ &= 1\cdot d\alpha^2 + W^2(\alpha)d\beta^2\\ &= g_1 d\alpha^2 + g_2 d\beta^2,\end{aligned}$$

$$\begin{aligned}\Delta &= \frac{1}{W}\left(\frac{\partial}{\partial\alpha}\left(W\frac{\partial}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{1}{W(\alpha)}\frac{\partial}{\partial\beta}\right)\right)\\ &= \frac{\partial^2}{\partial\alpha^2}+\frac{1}{W^2(\alpha)}\frac{\partial^2}{\partial\beta^2}.\end{aligned}$$

Thus, using (α,β), the wave equation and boundary conditions of the tapered plate problem are translated into a straight plate problem as follows:

$$\frac{1}{V_s^2}\frac{\partial^2 v}{\partial t^2} = \left(\frac{\partial^2}{\partial\alpha^2}+\frac{1}{W^2(\alpha)}\frac{\partial^2}{\partial\beta^2}\right)v, \quad (13)$$

$$v=0 \qquad \text{at} \quad \beta=\pm 1. \quad (14)$$

2.4 Separable solution

Considering fixed boundary conditions in (14), a separable solution,

$$v(\alpha,\beta) = f(\alpha)\cos\frac{\pi}{2}\beta\, e^{-j\omega t} \quad (15)$$

is assumed here. Because (13) is not a pure wave equation, a solution of a sinusoidal progressing wave along the x direction cannot be assumed. Therefore, as a general solution, $f(x)$ is assumed to satisfy

$$\frac{d^2 f(\alpha)}{d\alpha^2} = -k_x^2(\alpha)f(\alpha). \quad (16)$$

This solution represents a semi-sinusoidal progressing wave whose wavenumber k_x is not constant but a function of a position x.

2.5 Period equation of tapered plate

Substituting (15) into (13), we obtain a period equation of the tapered plate as follows by using (16).

$$\left(\frac{\omega}{V_s}\right)^2 = k_x^2(\alpha)+\left(\frac{\pi}{2W(\alpha)}\right)^2. \quad (17)$$

A plate width W_0 in the period equation of the straight plate (5) is changed into a tapered plate width $W(\alpha)$. A cut-off frequency (7) is also changed into

$$\omega_c(\alpha) = \frac{\pi V_s}{2W(\alpha)}, \quad (18)$$

which depends on the position α.

In Fig.2, a change of the wavenumber k along the x axis is shown. With decrease of the plate width from W_0 to W_1, a cut-off frequency increases from $\omega_0=\frac{\pi V_s}{2W_0}$ to $\omega_1=\frac{\pi V_s}{2W_1}$ according to (18). Thus, the dispersion curve is lifted up as shown in Fig.2 (b).

At last the cut-off frequency comes to be equal to the input frequency(Fig.(c)). After the wave passes the place where the width is $\frac{\pi V_s}{2\omega_{in}}$, the wavenumber becomes imaginary(Fig.(d)), and the wave is attenuated.

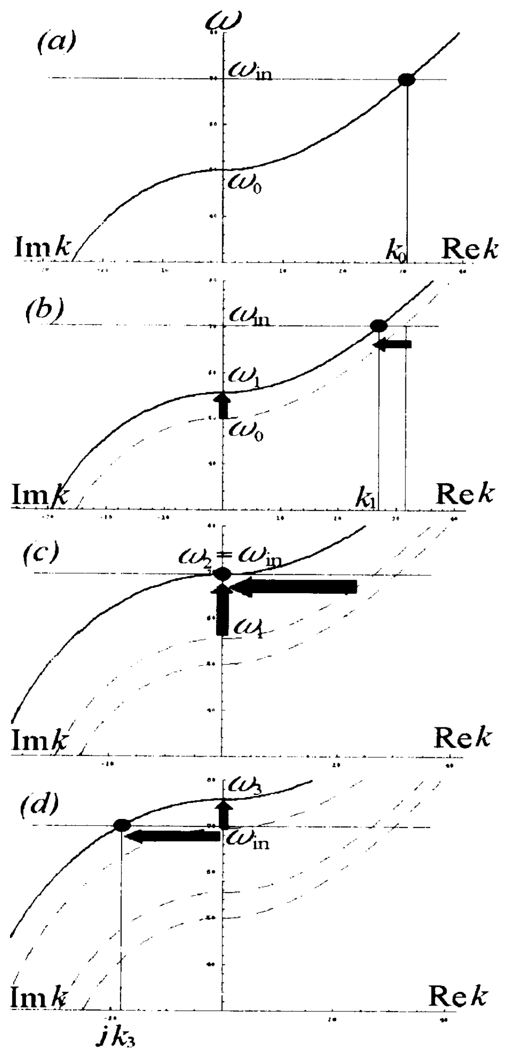

Figure 2: Change of a local wavenumber;k in the tapered plate

From the above discussion, a progressing wave in a tapered plate has a wavenumber $k_x(\alpha)$ and an amplitude $f(\alpha)$ which satisfies (16). Thus, the amplitude distribution of the wave in the tapered plate is decided by (16). We call the equation (16) the 'tapered plate equation'.

2.6 Relation between tapered plate equation and Schrödinger equation

Schrödinger equation for a stationary state in one dimension is represented as

$$\frac{d^2\phi(x)}{dx^2} = -k_x^2(x)\phi(x), \qquad (19)$$

$$k_x^2(x) = \frac{2m}{\hbar^2}(E - V(x)), \qquad (20)$$

where ϕ is a probability amplitude, E is an energy of the system, and $V(x)$ is a potential energy of the system. Schrödinger equation (19) and tapered plate equation (16) have the same shape. We change (17) into

$$k_x^2(\alpha) = \left(\frac{\omega}{V_s}\right)^2 - \left(\frac{\pi}{2W(\alpha)}\right)^2, \qquad (21)$$

and compare (21) with (20). Now, if we correspond the energy of a system E in the Schrödinger equation to an input energy in the tapered plate $\left(\frac{\omega}{Vs}\right)^2$, and if we correspond a potential energy $V(x)$ to $\left(\frac{\pi}{2W(\alpha)}\right)^2$, we can analyze the tapered plate equation by analogy to the Schrödinger equation. This analogy brings an important concept of potential energy of the tapered plate for elastic waves, where it is in inverse proportion to the plate width represented as $\left(\frac{\pi}{2W(\alpha)}\right)^2$. That is, the wider the plate width, the easier it is to pass through the plate for elastic waves.

The advantage of establishing the analogy between the tapered plate equation and the Schrödinger equation is that we can understand the property of the elastic waves easily by showing the graph of the potential energy of the plate and an input energy of the wave.

	(a)Schrödinger equation		(b)tapered plate equation
Input energy:	E	$\leftrightarrow$	$\left(\frac{\omega}{V_s}\right)^2$
Potential energy:	$V(x)$	$\leftrightarrow$	$\left(\frac{\pi}{2W(x)}\right)^2$

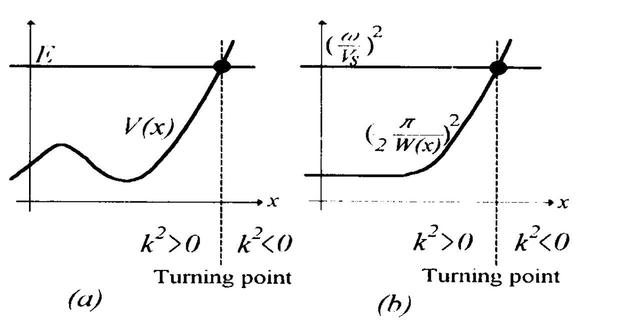

Figure 3: Analogy between (a) Schrödinger equation and (b) tapered plate equation

The property of the wave changes drastically at the place where the input energy is equal to the potential energy. This point is called a turning point(Fig.3) ,where $k_x(x) = 0$ for an input frequency ω. From (21), a signature of the square of a local wavenumber changes at the turning point. Where the input energy is lower than the potential, the wavenumber becomes imaginary, which results in an exponential damping of the wave. On the contrary, where the input energy is higher than the potential, the wavenumber becomes real, which results in the sinusoidal vibration.

Here, we represent a position of the turning point for an input frequency ω. When a plate width is assumed as

$$W(x) = W_0 e^{-dx}, \quad (22)$$

A local wavenumber is

$$k_x^2(x) = \left(\frac{\omega}{V_s}\right)^2 - \left(\frac{\pi}{2W_0}\right)^2 e^{-2dx}. \quad (23)$$

Therefore, the turning point defined as $k_x(x_0) = 0$ is obtained as

$$x_0 = \frac{1}{d}\log\frac{\pi V_s}{2W_0\omega}. \quad (24)$$

An amplitude distribution in the tapered plate is shown in the next section.

2.7 Amplitude distribution in a tapered plate

Now, assuming that the change rate of $W(x)$, therefore the change rate of the potential is small enough, the wavenumber is expanded around the turning point (24) as a linear function of x as follows:

$$\begin{aligned} k_x^2(x) &= k_x^2(x_0) + \frac{dk_x^2(x_0)}{dx}(x - x_0) \\ &= k^2(x - x_0), \end{aligned} \quad (25)$$

where

$$k^2 = 2d\left(\frac{\omega}{V_s}\right)^2. \quad (26)$$

After all, the tapered plate equation where the square of the local wavenumber is approximated as a linear function is

$$\frac{d^2 f(x)}{dx^2} + k^2(x - x_0)f(x) = 0. \quad (27)$$

Now, when the transformation of variables,

$$x - x_0 = X \quad (28)$$

$$f(x) = f(X + x_0) = g(X) \quad (29)$$

is conducted, we obtain

$$\frac{d^2 g(X)}{dX^2} + k^2 X g(X) = 0. \quad (30)$$

The solution of this equation is represented analytically using Bessel functions. (Appendix A)

A solution for $X > 0$ is

$$g(X) = \sqrt{X}\left(AJ_{\frac{1}{3}}\left(\frac{2}{3}kX^{\frac{3}{2}}\right) + BJ_{-\frac{1}{3}}\left(\frac{2}{3}kX^{\frac{3}{2}}\right)\right), \quad (31)$$

and a solution for the region $X < 0$ is represented as

$$g(X) = C\sqrt{|X|}K_{\frac{1}{3}}\left(\frac{2}{3}k|X|^{\frac{3}{2}}\right). \quad (32)$$

By determining coefficients A, B, C in order that $g(X)$ and $\frac{dg(X)}{dX}$ become continuous at $X = 0$, we obtain

$$g(X) = A\frac{\pi}{\sqrt{3}}\sqrt{X}\left(J_{\frac{1}{3}}\left(\frac{2}{3}kX^{\frac{3}{2}}\right) + J_{-\frac{1}{3}}\left(\frac{2}{3}kX^{\frac{3}{2}}\right)\right) \ (X > 0) \quad (33)$$

$$g(X) = A\sqrt{|X|}K_{\frac{1}{3}}\left(\frac{2}{3}k|X|^{\frac{3}{2}}\right) \quad (X < 0), \quad (34)$$

where A is determined by an input amplitude.

After all, by restoring variables with (28),(29), the amplitude distribution in the tapered plate is represented as follows, with a different expression before and after the turning point.

$$f(x) = A\frac{\pi}{\sqrt{3}}\sqrt{x - x_0}\left(J_{\frac{1}{3}}\left(\frac{2}{3}k(x - x_0)^{\frac{3}{2}}\right) + J_{-\frac{1}{3}}\left(\frac{2}{3}k(x - x_0)^{\frac{3}{2}}\right)\right) \quad (x > x_0) \quad (35)$$

$$f(x) = A\sqrt{|x - x_0|}K_{\frac{1}{3}}\left(\frac{2}{3}k|x - x_0|^{\frac{3}{2}}\right) \quad (x < x_0). \quad (36)$$

The amplitude distribution in the tapered plate is shown in Fig. 4.

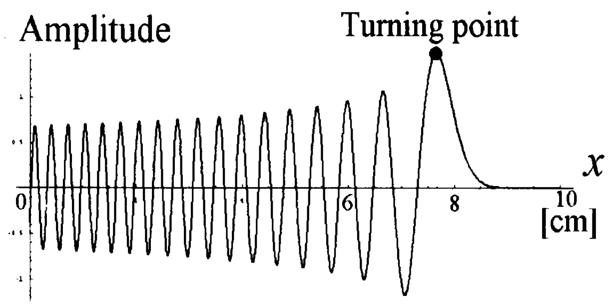

Figure 4: Amplitude distribution in the tapered Aluminum plate
--f =40[kHz], W_0 =2[cm], the wave is inputted at $x = 0$

3 Texture control using elastic waves in a tapered plate

The wave in the tapered plate shown in Fig.4 has a remarkable shape for controlling tactile sensation as follows.

1. From the place where the wave is inputted to the turning point, there exists a semi-sinusoidal wave. Beyond the turning point, the wave is attenuated and there is no vibration.
2. An amplitude increases gradually from the place where the wave is inputted, and becomes maximum at the turning point. The attenuation beyond the turning point is exponential.
3. In a linear elastic plate, several amplitude peaks can be generated by inputting waves with several frequencies.
4. A position of the turning point can be controled by an input temporal frequency. (Fig.5)

We discuss a method for controlling friction using the property 1,4 in Section 3.1, and a method for controlling texture using the property 2,3,4 in Section 3.2.

3.1 Control of friction

[2] showed that touching an object with ultrasonic vibration creates a tactile sensation of a slippery surface by an air lublication called squeeze effect. When an elastic wave is generated in a tapered plate, a region with vibration is bounded by an region without vibration. (property 1) A boundary between two regions is the turning point, and its position can be controlled by temporal frequency of the wave.(property 4) Thus the boundary of the slippery and the non-slippery region can be moved freely. Therefore a surface friction state of the plate is easily controlled.

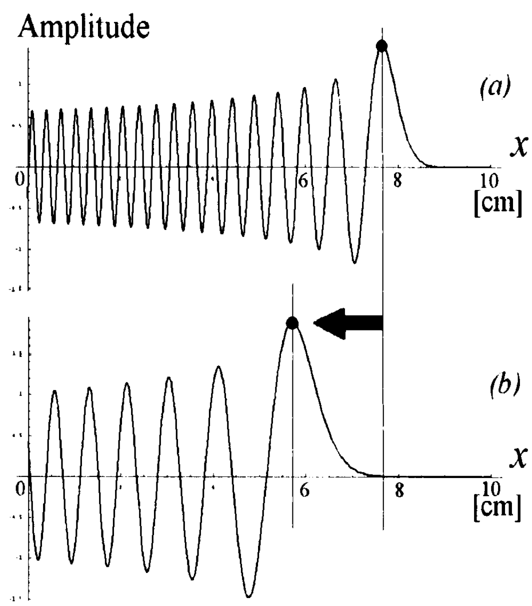

Figure 5: Control of the turning point by input frequency in an Aluminum tapered plate
(a)f=40[kHz] (b)f=20[kHz], W_0=2[cm]

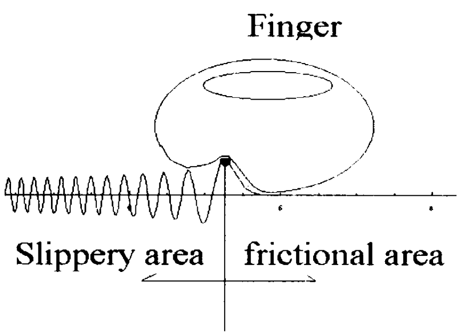

Figure 6: Control of friction

3.2 Control of Texture

Let us consider amplitude distributions of several types of elastic waves in a plate.

A progressing wave used for ultrasonic motor[6] has a constant amplitude over the plate. Thus, the amplitude distribution is flat. Even by a standing wave,

4.3 Future work

In a future work, we intend to investigate an tapered plate of aluminum, and vibrate it with ultrasonic frequency in order that we can control a frictional state and textures by elastic waves in the tapered plate. With higher frequency, an edge of the amplitude attenuation is steep. Therefore higher spatial resolution is obtained.

References

[1] Takaaki Nara, Taro Maeda, Yasuyuki Yanagida, Susumu Tachi "Tactile Display Using Elastic Waves", *Proc. IEEE VRAIS '98*, 43-50.

[2] Toshio Watanabe, Shigehisa Fukui: "A Method for Controlling Tactile Sensation of Surface Roughness Using Ultrasonic Vibration", *IEEE International Conference on Robotics and Automation*, 1995, 1134-1139

[3] L.M.Brekhovskikh: "Waves in Layered Media", *Applied Mathematics and Mechanics*, Academic Press, 1980

[4] W.M.Ewing, W.S.Jardetzky, F.Press: "Elastic Waves in Layered Media", McGraw-HILL, 1957

[5] E.O.J.Salbu, "Compressible Squeeze Films and Squeeze Bearing", *ASME Journal of Basic Engineering*, Vol.86, 1964, 355-366

[6] Minoru Kuribayahi, Sadayuki Ueha, Eiji Mori: "Excitation conditions of flexural traveling waves for a reversible ultrasonic linear motor", *J. Acoust. Soc. Am. 77(4)*, April 1985, 1431-1435

Appendix : Relation between the tapered plate equation and the Bessel differential equation

A Bessel differential equation

$$\frac{d^2u}{dz^2} + \frac{1}{z}\frac{du}{dz} + \left(1 - \frac{n^2}{z^2}\right)u = 0, \qquad (37)$$

is transformed into

$$\frac{d^2v}{dz^2} + \frac{1-2a}{z}\frac{dv}{dz} + \left(b^2c^2x^{2c-2} + \frac{a^2-n^2c^2}{z^2}\right)u = 0, \qquad (38)$$

by the transformation of variables,

$$z = bx^c \qquad (39)$$

$$u = x^{-a}v. \qquad (40)$$

Now, when we set each constant as

$$a = \frac{1}{2},\ c = \frac{3}{2},\ b = \frac{2k}{3},\ n = \frac{1}{3}, \qquad (41)$$

a tapered plate equation with a real wavenumber is obtained:

$$\frac{d^2v}{dz^2} + k^2xv = 0. \qquad (42)$$

A general solution for the Bessel equation is

$$v = AJ_\mu(z) + BJ_{-\mu}(z). \qquad (43)$$

Thus, by the transformation of variables, a general solution of the tapered plate equation is

$$v = \sqrt{x}\left(AJ_{\frac{1}{3}}\left(\frac{2}{3}kx^{\frac{3}{2}}\right) + BJ_{-\frac{1}{3}}\left(\frac{2}{3}kx^{\frac{3}{2}}\right)\right) \qquad (44)$$

In a similar manner, the tapered plate equation with an imaginary wavenumber

$$\frac{d^2v}{dx^2} - k^2xv = 0 \qquad (45)$$

is obtained from the modified Bessel equation

$$\frac{d^2u}{dz^2} + \frac{1}{z}\frac{du}{dz} - \left(1 + \frac{n^2}{z^2}\right)u = 0 \qquad (46)$$

by the same transformation of variables. Using a solution of the modified Bessel equation which becomes zero when $|x| \to \infty$;

$$v = CK_\mu(z), \qquad (47)$$

a solution of the tapered plate is

$$v = C\sqrt{|x|}K_{\frac{1}{3}}\left(\frac{2}{3}k|x|^{\frac{2}{3}}\right), \qquad (48)$$

where K is a modified Bessel function.