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DIRECT CAMERA CALIBRATION USING TWO CONCENTRIC CIRCLES FROM A SINGLE VIEW

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Abstract

This paper describes a new camera calibration method based on ellipses properties. We demonstrate that the calibration is possible using one view of two concentric circles of known radii. Based on the estimation of the projected circle centers and on the ellipses properties as the perspective projection of circles, our algorithm estimates both the focal length and the pose and orientation of the camera.
We validate the performances of our algorithm using both synthetic and real images. The use of circles greatly simplifies the calibration problem. A noise analysis is also made to compare the accuracy of the results with another well-known calibration method.
This calibration method is also well-suited to the 3D-reconstruction problem from an image sequence of an object placed on a turn-table.

Keywords : Camera Calibration, Ellipse properties, Projective Geometry, 3D Reconstruction, Virtual Reality.

1. Introduction

Considerable efforts have been made to recover photorealistic models of real objects. The most important stage of the modeling is the calibration stage. Camera calibration is to estimate the intrinsic and extrinsic parameters of the camera. By intrinsic, we mean internal parameters of the camera : focal length and principal point, assuming zero skew, unit aspect ratio, no distortions and square pixels, which are good assumptions for modern cameras. By extrinsic parameters, we mean the pose and orientation of the camera in an absolute frame.
Calibration methods are divided into two categories : calibration using calibration pattern [2] [6] (a 3D object of known geometry with a known position in space), and self-calibration methods [8] (absolute quadric estimation over a sequence of images). When using a calibration pattern, the precision of the parameter estimation is related to the accuracy of the image measurements of the calibration pattern, and the algorithm needs a large set of image points to converge using a non-linear optimization [1].
In our method, we propose to calibrate the camera using only one image of two concentric circles of known radii. We first characterize the perspective projection of a circle, which is an ellipse in a classic configuration (non degenerated conic). The ellipses are detected automatically using the Hough transform based algorithm [9]. The coefficients of the conic (eg. the ellipse) are estimated using standard conic fitting algorithm [3] [7]. Looking at the coefficients of the ellipse, they are function of intrinsic and extrinsic parameters as well as the projected circle center [4] (which is different from the ellipse center). To estimate the projected circle center we use projective geometry tool like the cross-ratio [5]. Then from the N image points of the estimated ellipse, its six coefficients (coming from the conic equation) and the projected circle center, this leads to an over-determined system for which the solution gives the focal length, the position of the camera and the normal to the surface supporting the 3D circle.

Our method using both synthetic and real images, and a noise analysis gives comparable performance compare to other camera calibration method using a complex calibration pattern.
This paper is organized as follows : in section 2, we present the pinhole camera model which is derived from projective geometry. In section 3, we derive the equation of the perspective projection of a circle and how to estimate the projected circle center. In section 4, we present the core of our calibration method based on ellipse equations and properties. In section 5, we show the results of the calibration algorithm on synthetic and real images, and a noise analysis. Finally, we conclude this paper in section 6.

2. Camera Model

As it is known in the 3D vision community, the most commonly used model to represent a vision sensor, namely a camera, is the pinhole model (see Figure 1) :

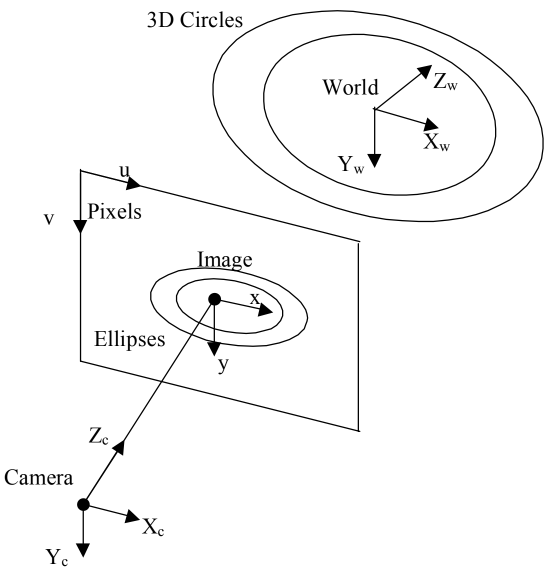

Figure 1 : Pinhole Camera Model and Circle Calibration Pattern.

Let the origin of the world frame be located at the center of the 3D concentric circles. The Y_w-Axis is orthogonal to the 3D circle plane. The coordinates of a circle point is expressed as the vector :

$$\mathbf{M} = \begin{bmatrix} X_w & 0 & Z_w & 1 \end{bmatrix}^T \tag{1}$$

The corresponding image point is :

$$\mathbf{m} = \begin{bmatrix} u & v & 1 \end{bmatrix}^T \tag{2}$$

The transformation between the image points and the world points is the well-known perspective projection relation :

$$\lambda.\mathbf{m} = \mathbf{P}.\mathbf{M} \tag{3}$$

Where

$$\mathbf{P} = \mathbf{A}.\begin{bmatrix} \mathbf{R} & \mathbf{T} \end{bmatrix} \tag{4}$$

$\mathbf{P}$ is projection operator, λ is a scale factor due to homogeneous coordinate system. The matrix **A** depends only on the intrinsic parameters of the camera :

$$\mathbf{A} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

We assume that the sensor is perfect : zero skew, unit aspect ratio, no distortions and square pixels.

R and **T** are respectively the rotation matrix and the translation matrix (the extrinsic parameters) of the sensor regarding to the world reference frame.

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \tag{6}$$

$$\mathbf{T} = \begin{bmatrix} Tx \\ Ty \\ Tz \end{bmatrix} \tag{7}$$

3. Ellipses as Circles Perspective Projection

The calibration pattern we are using is composed with only two concentric circles with known radii. The big circle has radius R1 (which we will call the principal circle) and the small circle has radius R2.

3.1. Circle Perspective Projection

Using the projection operator $\mathbf{P}$, the image points are given by :

$$\begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_w \\ 0 \\ Z_w \\ 1 \end{bmatrix} \tag{8}$$

The principal circle is in the plane $Y_w=0$. The locus of image points describing the circle is defined by :

$$\begin{cases} u = \dfrac{p_{11}X_w + p_{13}Z_w + p_{14}}{p_{31}X_w + p_{33}Z_w + p_{34}} \\ v = \dfrac{p_{21}X_w + p_{23}Z_w + p_{24}}{p_{31}X_w + p_{33}Z_w + p_{34}} \end{cases} \tag{9}$$

By extracting X_w and Z_w, we have :

$$\begin{cases} X_w = A_1 u + B_1 v + C_1 \\ Z_w = D_1 u + E_1 v + F_1 \end{cases} \tag{10}$$

Where A_1, B_1, C_1, D_1, E_1 and F_1 depend on the intrinsic and extrinsic camera parameters. In the world reference frame, the principal circle, centered at the origin, has the following equation :

$$C(X_w, Z_w) = X_w^{\ 2} + Z_w^{\ 2} - R1^2 = 0 \tag{11}$$

Therefore by using (1) in (2) we obtain the principal ellipse,

$$C'(u,v) = Au^2 + Buv + Cv^2 + Du + Ev + F = 0 \tag{12}$$

This is the equation of a conic. An ellipse is obtained with the constraint $B^2 - 4AC < 0$. In practice, this constraint is usually satisfied if data are not all situated in a flat section and the conic is not degenerated. The coefficients of the conic are estimated using an ellipse fitting algorithm [3].

3.2. Estimation of the Projected Circle Center

As stated in [4], [5], the projected circle center is different from the ellipses centers. The method to determine the projected circle center is inspired by [5].
[5] define a new projective invariant for the projection of concentric circles : "the center of projected concentric circles always lies on a line defined by the ellipses centers under any projective transformations" (see Figure 2).

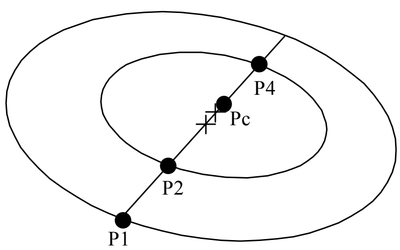

Figure 2 : Projected circle center is on a line defines by the centers of the two ellipses.

The only projective invariant is the cross-ratio of four aligned points. In our case, we can define the following cross-ratio (see Figure 2) :

$$Cr2D = \frac{\Delta P1Pc \cdot \Delta P2P4}{\Delta P1P4 \cdot \Delta P2Pc} \tag{13}$$

Where

$$\Delta PiPj = \sqrt{(XPi - XPj)^2 + (YPi - YPj)^2} \tag{14}$$

We have also

$$Cr3D = \frac{2R1}{R1 + R2} \tag{15}$$

Since the cross-ratio is invariant, we can set :

$$Cr2D = Cr3D \tag{16}$$

Therefore it is possible to extract the image coordinates Xc and Yc of the projected circle center Pc.

4. Camera Calibration

Since the coefficients of the principal ellipse have been estimated, we can define it with its matrix form as defined by [4] :

$$\mathbf{Q} = \begin{bmatrix} A & B/2 & D/2f \\ B/2 & C & E/2f \\ D/2f & E/2f & F/f^2 \end{bmatrix} \tag{17}$$

As expressed in [4], the unit normal to the circle plane is computed as follows :

$$\begin{bmatrix} R_{12} \\ R_{22} \\ R_{32} \end{bmatrix} = \pm N \mathbf{Q} \begin{bmatrix} Xc \\ Yc \\ f \end{bmatrix} \tag{18}$$

Where $N[\cdot]$ designates normalization to a unit vector and Xc , Yc are the image coordinates of the projected circle center. Therefore we can extract each component of the unit normal :

$$\begin{cases} R_{12} = \dfrac{\alpha_1}{norme} \\ R_{22} = \dfrac{\alpha_2}{norme} \\ R_{32} = \dfrac{\alpha_3}{f \cdot norme} \end{cases} \tag{19}$$

Where :

$$norme = \sqrt{\alpha_1^2 + \alpha_2^2 + \frac{\alpha_3^2}{f^2}} \tag{20}$$

and

$$\begin{cases} \alpha_1 = \dfrac{2A \cdot Xc + B \cdot Yc + D}{2} \\ \alpha_2 = \dfrac{B \cdot Xc + 2C \cdot Yc + E}{2} \\ \alpha_3 = \dfrac{D \cdot Xc + E \cdot Yc + 2F}{2} \end{cases} \tag{21}$$

We can also express the following relations of the translation vector :

$$Ty = \frac{Yc \cdot Tz}{f} \qquad Tx = \frac{Xc \cdot Tz}{f} \tag{22}$$

Using (19), (20), (21), (22) in (12), we obtain the new coefficients for the principal ellipse :

$$\begin{cases} A_c = A_{c1} \cdot f^2 + A_{c2} \dfrac{f^2}{Tz^2} + A_{c3} \\ B_c = B_{c1} \cdot f^2 + B_{c2} \dfrac{f^2}{Tz^2} + B_{c3} \\ C_c = C_{c1} \cdot f^2 + C_{c2} \dfrac{f^2}{Tz^2} + C_{c3} \\ D_c = D_{c1} \cdot f^2 + D_{c2} \dfrac{f^2}{Tz^2} + D_{c3} \\ E_c = E_{c1} \cdot f^2 + E_{c2} \dfrac{f^2}{Tz^2} + E_{c3} \\ F_c = F_{c1} \cdot f^2 + F_{c2} \dfrac{f^2}{Tz^2} + F_{c3} \end{cases} \tag{23}$$

Therefore we have :

$$C'(u,v) = A_c u^2 + B_c uv + C_c v^2 + D_c u + E_c v + F_c = 0 \tag{24}$$

For the N points of the ellipse, this equation can be used to form the following over-determined system :

$$WX = B \tag{25}$$

Where

$$\begin{bmatrix} W_{11} & W_{21} \\ \vdots & \vdots \\ W_{1N} & W_{2N} \end{bmatrix} \begin{bmatrix} f^2 \\ \dfrac{f^2}{Tz^2} \end{bmatrix} = \begin{bmatrix} B_1 \\ \vdots \\ B_N \end{bmatrix} \tag{26}$$

The solution of this system is obtained as the least square pseudo-inverse technique :

$$X = \left(W^T W\right)^{-1} W^T B \tag{27}$$

From X, we obtain the focal length f and Tz. Using (19), (20), (21), and (22), we can calculate the extrinsic parameters.

It is important to remark that we only obtain the second column of the rotation matrix, which is the unit normal to the circle supporting plane. If we want to obtain the two other columns, we can use two circles situated on two orthogonal planes (see Figure3) and using the algorithm in [4], this leads to the whole rotation matrix.

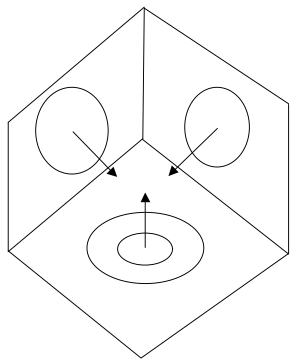

Figure 3 : Final calibration pattern to obtain the entire rotation matrix.

5. Camera Calibration Experimental Results

Experiments with synthetic images and real images have been carried out. Ellipses are fitted by a direct least square method of Fitzgibbon [3].

5.1. Synthetic Images

For synthetic images, we used the following simulation parameters :

f	3000 pixels
R_{12}	-0.07
R_{22}	0.93
R_{32}	0.34
Tx	-4.3 mm
Ty	41 mm
Tz	750 mm
R1	50 mm
R2	28 mm

To find out the noise robustness of our calibration algorithm, we added noise in pixels to the ellipse data.
The results of the experiment are given in the following figures :

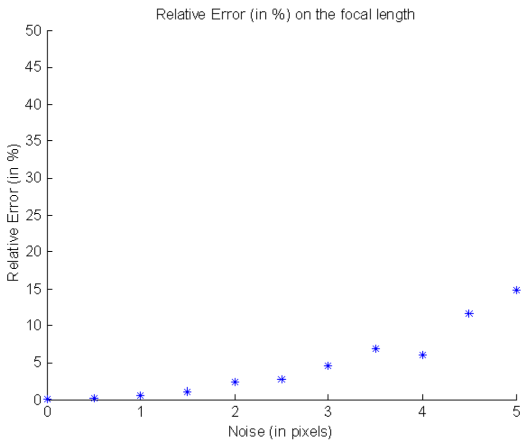

Figure 4 : Relative error (in %) on the focal length.

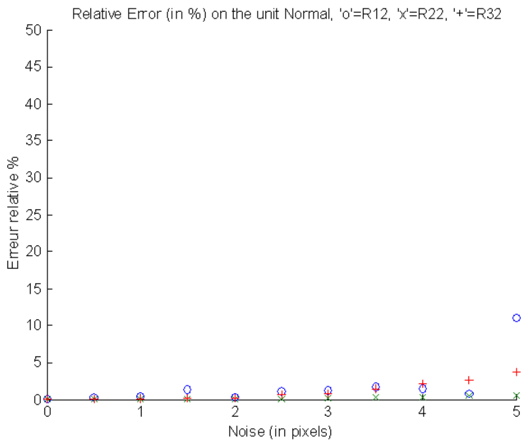

Figure 5 : Relative error (in %) on the unit normal.

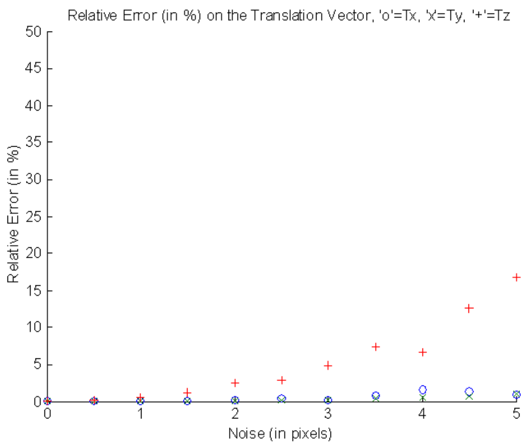

Figure 6 : Relative error (in %) on the translation vector.

5.2. Real Images

The calibration pattern used is shown in Figure 7. The images were taken by a digital camera Fujifilm FinePix 2600 Zoom. The images have a resolution of 1280x960. The ellipses were detected using [9].

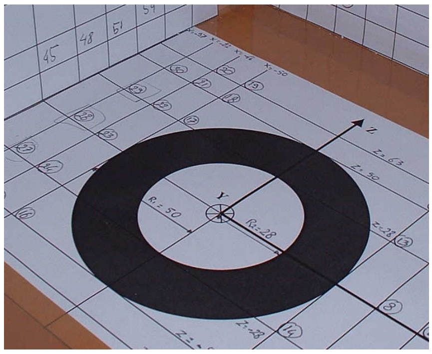

Figure 7 : Calibration Pattern.

For performance comparison, the system was also calibrated with the well-known calibration algorithm from Tsai[2].

	Tsai[2]	**Proposed**
f	**2043.2**	**2131.8**
R_{12}	**0.0009**	**0.001**
R_{22}	**0.83**	**0.82**
R_{32}	**0.55**	**0.564**
Tx	**-2.16**	**-2.11**
Ty	**35.3**	**35.6**
Tz	**393.2**	**400.7**

Using the calibration parameters obtained with our algorithm, it is possible to re-drawn the projection of the principal circle (see Figure8) :

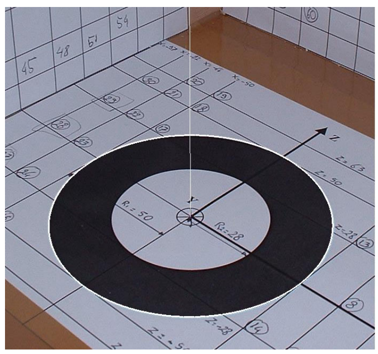

Figure 8 : Re-projected principal circle (in white) and the normal (in white).

6. Conclusion

In this paper, we have presented a new camera calibration method based on the ellipses properties. Using only a single view of two concentric circles, an estimation of the projected circle center and an ellipse fitting algorithm, we shown that it is possible to obtain the focal length, the pose and the orientation of the camera.

With synthetic and real images, our algorithm shows comparable performances with the existing calibration algorithm, but with only one image and two concentric circles.

This calibration method is also well-suited to the 3D-reconstruction problem from an image sequence of an object placed on a turn-table, using that one as the calibration pattern.

An extension of our algorithm, will be to integrate an estimation of the principal point, the skew and the distortions of the sensor.

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