

# Rendering Methods for Models With Complicated Micro Structures

Shun Iwasawa\*, Naohiro Shichijo\*\*, Yoichiro Kawaguchi\*\*

\*Graduate School of Interdisciplinary Information Studies, The University of Tokyo

\*\* Interfaculty Initiative in Information Studies, The University of Tokyo

7-3-1, Hongo, Bunkyo-ku, Tokyo, 113-0033 JAPAN

{qq46104, shichi, yoichiro}@iii.u-tokyo.ac.jp

## Abstract

The male *Morpho* Butterflies have peculiar vivid cobalt blue wings caused by microscopic structures on its scales. Its color is due to structural specificity and not caused by its pigments. Those reflectional color effects are known as “structural colors”.

The methods to represent structural colors have been extensively studied from the viewpoint of anisotropic reflection, but those previous studies were based on phenomenological modeling of measured reflectance functions and thus specific microstructure of the target surfaces was disregarded.

In this paper, a physically sound structural model is proposed taking account of the effect of interference, diffraction, light scattering, anisotropy and semi-periodicity of microstructure on its surface and a new rendering method is developed.

**Keywords:** structural colors, full spectral rendering, Morpho butterflies

## 1. Introduction

“Structural color” or “iridescence” is defined variously in different contexts. According to Kinoshita’s definition [1], structural color is general term for the color phenomena that come from the difference in the behaviors of lights of various wavelengths, and is separated from the normal color phenomena by pigments.

In the history of computer graphics researches, there are very few studies involving or representing the structural colors. Almgren et al.[7] developed the shading algorithms of soap film interference, and surveyed results on the geometry of bubble clusters for visualizing photorealistic computer graphics of soap bubbles. Nishita et al.[8] proposed a model on streaks of light taking into account both refraction and diffraction of light aiming at drive simulators. Gondek et al. [9] developed a virtual goniospectrophotometer by using a Monte Carlo ray tracer to cast rays into a surface for analyzing and making pictures of thin films, idealized pigmented materials, and pearlescent paints. Nishita et al.[10] and Preetham et.al. [11] developed an analytic

model for rendering sunlight and skylight considering the effects of atmosphere. Nagata et al. [12] proposed a method of modeling and visualizing pearls for a pearl-quality evaluation simulator, by using a physical model, called an “illuminant model,” for multilayer film interference considering the multiple reflection in spherical bodies. Stam [13] developed a reflection models for metallic surfaces that handle the effects of diffraction.

At the whole, relatively simple phenomenon such as the thin film interference has been extensively studied, and is enough to be represented in real-time [2]. On the other hand, other phenomenon, especially multilayer thin film interference, diffraction, Mie scattering and Rayleigh scattering are attracting increasing attention recently. However, it is still controversial that the representation of the colors of the surfaces with the complicated and semi-periodic micro structures, as in surfaces of physical being such as *Morpho* butterflies.

The *Morpho* Butterflies (Fig.1)[14] are some of the well-known iridescent butterflies mainly living in Amazonian watershed of South America.

Until now, many researchers have long been tried to investigate these vivid cobalt blue colors. By researches using images with scanning electron microscope since 1920s, it was found that these iridescences were caused by the complicated microstructures on its surfaces [1].

Figure.2 and 3 show the SEM image of the cross-sectional surface of a typical scale covering over the wings of male *Morpho rhetenor*. Each scale is about 150x70μm in its size, and is composed of periodic vertical ridges separated by 700~800nm (Fig.2). Each ridge forms tree-like structures, and has 8~10 shelf-like cuticle (refractive index is around 1.56) layers (Fig.3). Each cuticle layers are separated by about 200nm. These semi-periodic complicated microstructures contribute iridescence property in scales. And also, the columnar arrangement of ridges results strong anisotropy in iridescence property in scales.



Fig.1 Photograph of the male *M. rhetenor*. [14]

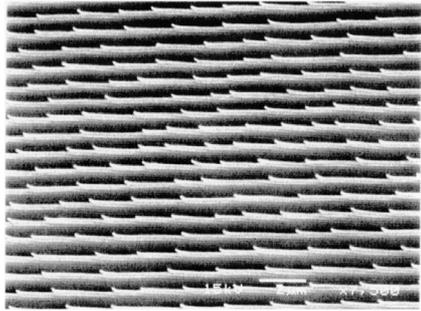


Fig.2 Oblique view of the scanning electron microscope images of the ground scale of *M. didius* [3].

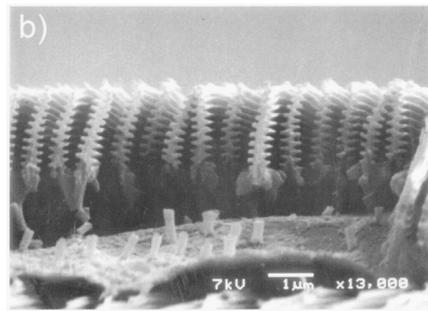


Fig.3 Scanning electron microscope image of the cross section of the iridescent scale of *M. rhetenor*. [3]

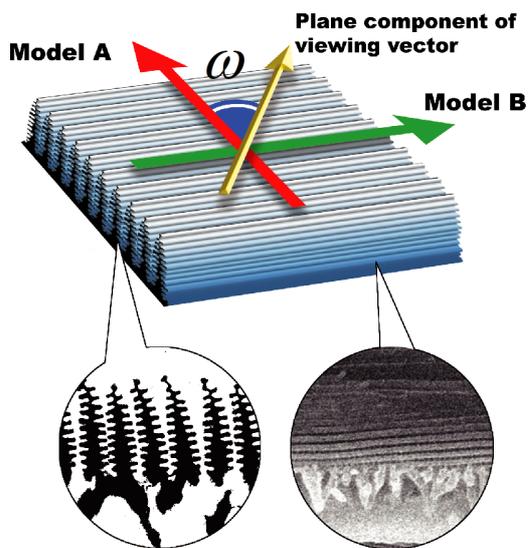


Fig.4 Combining two models to achieve the anisotropy of scale.

## 2. Exposition

### 2.1. Development of micro structural model

By examining the structure described in the previous section, analytical solution of BRDF by the full three-dimensional treatment found to be infeasible because of its complicated three-dimensional structure. Thus we propose a pseudo three dimensional analytic model of iridescence property by incorporating two different models. Firstly (Model A in Figure 4), the scale is treated as repeated tree-like structures. Secondly (Model B in Figure 4), the scale is treated as multilayer thin film. By incorporating those two models, an approximate full three-dimensional BRDF is obtained.

### 2.2. Model A: the separate lamellae model

The cross-section of the scale with a plane perpendicular to the “Model B” vector is composed of periodical tree-like structures (as seen in left inset of Figure 4). Kinoshita et al. [3] proposed an analytical model of iridescence function of Morpho wings with following assumptions, 1) the lamellae (branch) has finite length and separated by air layer with each adjacent lamellar (of different trees), 2) each tree is elevated randomly in vertical direction and the amount of elevation is not correlated (Figure 5).

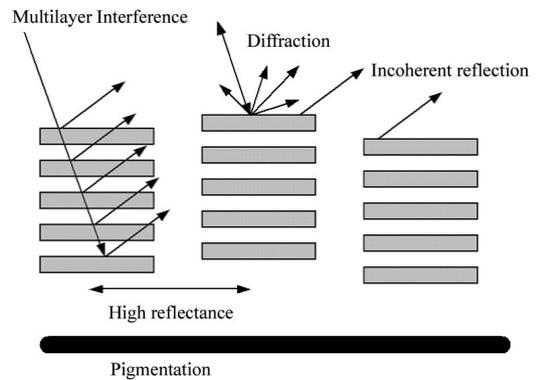


Fig.5 Principal contributions to iridescence function in “the separate lamellae model”

To elucidate the iridescence function of the Morpho wing, Kinoshita et al. [3] proposed an analytic model in which a plane wave incident on  $N$  separate lamellae having a finite width in one direction and an infinite length in the other. Further, each cuticle layer is assumed to be infinitely thin, and the incident and diffracted light is not subjected to reflection nor refraction while passing through the other layers (Fig. 6). The light diffracted by each layer interferes in the far-field region.

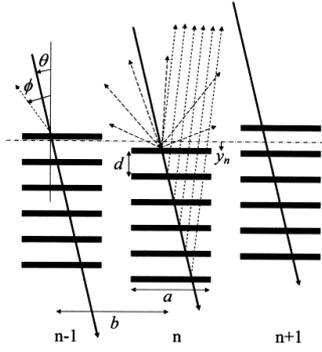


Fig. 6 A simple model for the lamellar structure on the scale. A plane wave is incident on N separate lamellae with different heights [3].

If there is no spatial correlation among ridge heights, the intensity of the diffracted light toward the direction of  $\phi$  under the illumination of the incident light from the direction of  $\theta$  is expressed as

$$I_{\phi} = \frac{a^2}{2} \cdot \frac{\sin^2(kdvM/2)}{\sin^2(kdv/2)} \cdot \frac{\sin^2(kau/2)}{(kau/2)^2} \cdot N \cdot I_{\theta} \cos^2 \theta,$$

where  $d$ ,  $a$ ,  $b$ , and  $M$  are the interval of cuticle layers, the width of a cuticle layer, the ridge separation and the number of cuticle layers, respectively.  $I_{\theta}$  is the intensity of incident light per unit area and  $k = 2\pi/\lambda$  with the wavelength of light  $\lambda$ . The factors  $u$  and  $v$  are expressed as

$$\begin{aligned} u &= \sin \theta + \sin \phi \\ v &= \cos \theta + \cos \phi. \end{aligned}$$

See Kinoshita et al. See Kinoshita et al. [2002] for exact solution of this model.

### 2.3. Model B: the inclined and curved multilayer model

The cross-section of the scale with a plane perpendicular to the side of “Model A” is considered as multilayer thin films with two alternate layers: cuticle and air. Layers are slightly inclined upward from the inside to the outside of wing. And also, slightly curved irregularly (Fig.7). The irradiance function of multilayer thin films is solved analytically and represented as recurrence formula as shown in Ishiguro et. al. [4].

First, we assume the reflectivity of light, in case of the  $p-1$  layers are being piled up on the base layer with the refractive index  $n_0$ , and the light incidences with the angle  $\theta_p$  from the  $p$ -th medium on the layers with the refractive index  $n_p$ . The reflectivity of light are being assumed as  $R_{p,0}$ . Then, when the new ( $p+1$ -th) medium putted on the  $p$ -th medium as shown in (Fig.8), the new reflectivity of light  $R_{p+1,0}$  expressed as following recurrence formula.

$$R_{p+1,0} = \frac{R_{p+1,p} + R_{p,0} + 2\rho_{p+1,p} \sqrt{R_{p,0}} \cos(\gamma_{p,0} + \gamma_p)}{1 + R_{p+1,p} R_{p,0} + 2\rho_{p+1,p} \sqrt{R_{p,0}} \cos(\gamma_{p,0} + \gamma_p)},$$

with

$$\begin{aligned} \gamma_p &= \frac{2\pi}{\lambda} \Delta_p \\ \Delta_p &= 2n_p d_p \cos \theta_p \end{aligned}$$

where  $d_p$  is the thickness of  $p$ -th layer (medium).  $\gamma_{p,0}$  is the discrepancy of the phase of reflection light and is expressed as

$$\tan \gamma_{p,0} = \frac{\sqrt{R_{p-1,0}}(1 - \rho_{p,p-1}^2) \sin(\gamma_{p-1,0} + \gamma_{p-1})}{\rho_{p,p-1}(1 + R_{p-1,0}) + \sqrt{R_{p-1,0}}(1 + \rho_{p,p-1}^2) \cos(\gamma_{p-1,0} + \gamma_{p-1})},$$

where  $\rho_{p+1,p}$ ,  $R_{p+1,p}$  are the reflectivity of amplitude and of energy of light, and are expressed as

$$\begin{aligned} \rho_{p+1,p} &= \frac{Y_p - Y_{p+1}}{Y_p + Y_{p+1}}, \\ R_{p+1,p} &= |\rho_{p+1,p}|^2, \end{aligned}$$

When the reflectivity of light energy is regarded as the mean between the both reflectivity of light energy of polarized lights,  $Y_i$  can be expressed as

$$Y_i = \left( n_i \cos \theta_i + \frac{n_i}{\cos \theta_i} \right) / 2, \quad i = p, p+1$$

The incidence angle  $\theta_p$  upon the each layer is expressed in two ways.

1), when the  $p$ -th layer is air,

$$\theta_p = \theta - \delta$$

2), when the  $p$ -th layer is cuticle, through the Snell's law,

$$\sin \theta_p = \sin(\theta - \delta) \cdot n_{cuticle} / n_{air}$$

where  $\delta$  is the average of the gradient of multilayer.

Considering the diffraction by the curvature of multilayer, reflectance intensity  $I_{\phi}$  with reflection angle  $\phi$ , is expressed as

$$\text{when } |\Delta\phi| < \phi_{\alpha}, \quad I_{\phi} = \cos\left(\frac{\Delta\phi}{\phi_{\alpha}} \cdot \pi/2\right) \cdot I_{\phi_0}$$

where  $\Delta\phi$  is the declination between  $\phi$  and the specular reflection angle  $\phi_0$ , and  $I_{\phi_0}$  is the reflectance intensity of the reflection light with the specular reflection angle.

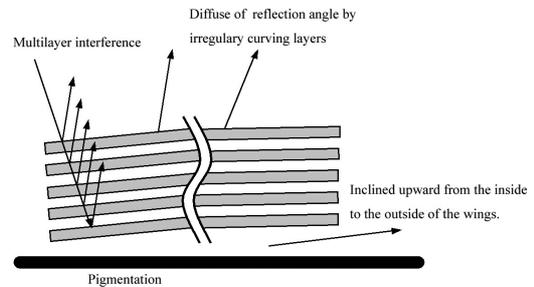


Fig.7 Principal contribution to iridescence function in “the inclined and curved multilayer model”

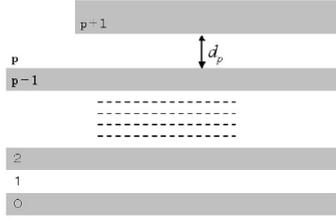


Fig. 8 Putting the new p+1-th medium on the p layers.[4]

## 2.4. Blending Two Models

To calculate irradiance function  $I$  from arbitrary viewing

angles  $\omega$ , the irradiance function of these two models ( $I_A, I_B$ ) are blended by the following equation

$$I = \{I_A + I_B + (I_A - I_B) \cos 2\omega\} / 2 .$$

## 3. Implementation

From the models as described above, spectral reflectance of each elevation incidence angle  $\theta$ , each elevation reflectance (viewing) angle  $\phi$  and each angle of anisotropy  $\omega$  are obtained.

Reflectance intensities are calculated in the 380nm to 740nm range, every 10nm apart. For the spectral intensity of incidence light, We applied CIE standard illuminant D65 (Fig.9)[5], which is intended to represent average daylight and has a correlated color temperature of approximately 6500 K.

The irradiance function is converted to the RGB color intensity using CIE1931 color-matching function (Fig.10)[6] which provides a relative measure of the ratios of the three primaries needed to match each spectrum color.

Thus, the RGB parameters of reflection light are derived as

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = k \cdot \begin{pmatrix} \sum_{\lambda=380nm}^{740nm} I_{\lambda} \cdot r(\lambda) \\ \sum_{\lambda=380nm}^{740nm} I_{\lambda} \cdot g(\lambda) \\ \sum_{\lambda=380nm}^{740nm} I_{\lambda} \cdot b(\lambda) \end{pmatrix},$$

where  $r(\lambda), g(\lambda), b(\lambda)$  and  $I_{\lambda}$  are the color matching functions for three primaries and computed reflectance intensity of some wavelength of light  $\lambda$ .  $k$  is scalar.

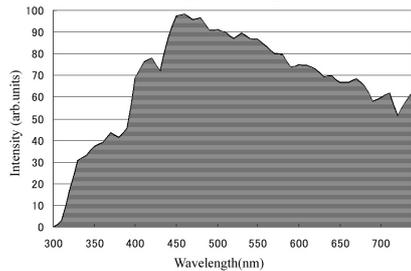


Fig. 9 spectrum intensity of CIE standard illuminant D65.[5]

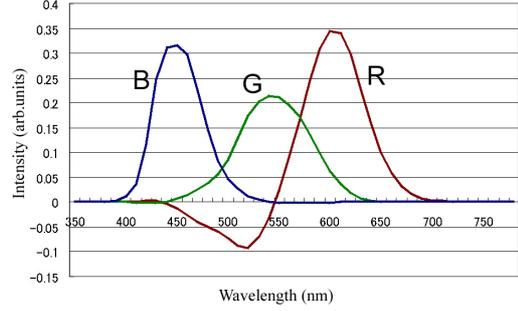


Fig. 10 CIE1931 color-matching function.[6]

## 3.1. Rendering

For Rendering, we applied Blue Moon Rendering Tools (BMRT) which are 3D graphics APIs for rendering with ray tracing, and conforming to PIXAR's RenderMan Interface. We put some data as follows: 1) Precomputed three dimensional array of the reflectance color with each discrete angle,  $\theta, \phi$  and  $\omega$ .  $\omega$  is computed by using "Anisotropy angle map" described in 4), 2) Simple geometry data such as square polygon, 3) Displacement map, and 4) Anisotropy angle map.

### 3.1.1. Displacement map

An actual *Morpho* Butterflies' wing has the small corrugations in a radial pattern. They make the normal vectors fluctuated, and might be one of the strong factors of glittering effect. So, we applied the image as (Fig.11) for displacement map to represent these corrugations.

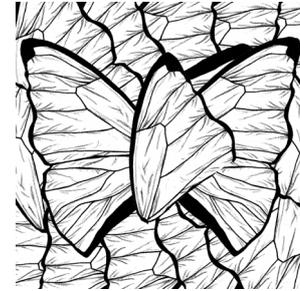


Fig. 11 image for displacement map.

### 3.1.2. Anisotropy angle map

The scales on the wings of *Morpho* Butterflies are paved with radial alignment out from the center of whole bodies. "Anisotropy angle map"(Fig.12) is to define the horizontal angle of the scale by contrasting distribution. An angle  $\omega_{map}$  between the upward direction of "Anisotropy angle map" and horizontal direction of the ridge ("Model B" vector in Fig.4), on the any point on the "Anisotropy angle map" with the brightness  $Y$ , can be obtained as

$$Y = \omega_{map} / 2\pi .$$



Fig. 12 image for “Anisotropy angle map”.

## 4. Results and Discussion

### 4.1. Other parameters

Table. 1 describes other parameters for the simulation.

Table. 1 Parameters for the simulation.

(A) Parameters for “the separate lamellae model”.

Data items	Symbols	Values
width of a cuticle layer	a	300(nm)
interval of cuticle layers	d	235(nm)
number of cuticle layers	M	9
number of tree-like structures	N	50

(B) Parameters for “the inclined and curved multilayer model”

Data items	Symbols	Values
thickness of the air layer	d <sub>air</sub>	135(nm)
thickness of cuticle layer	d <sub>cuticle</sub>	100(nm)
refractive index of the air	n <sub>air</sub>	1
refractive index of cuticle	n <sub>cuticle</sub>	1.56
average of the gradient of multilayer	$\delta$	2.5 (degree)
diffusion width by curvature of multilayer	$\phi_\alpha$	0.5 (radian)

### 4.2. Angular dependence of the reflectivity

(Fig.13) and (Fig.14) are the comparison between angular dependence of the reflectivity of the actual (Fig.13)[3], and the simulation result (Fig.14) of *Morpho* butterfly scale to the light of various wavelength. The light of 480nm wavelength reflects with the strongest intense and the wing shows cyan color, which roughly corresponds to the real iridescent characteristics of *Morpho* wings.

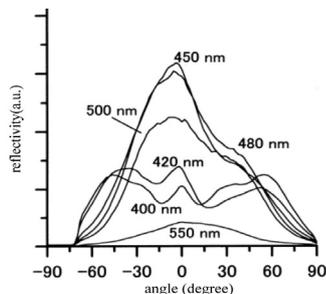


Fig. 13 Angular dependence of reflected light intensity in a plane perpendicular to the ridges for various wavelengths from a wing without cover scales of the male *M. didius* under normal incidence.[3]

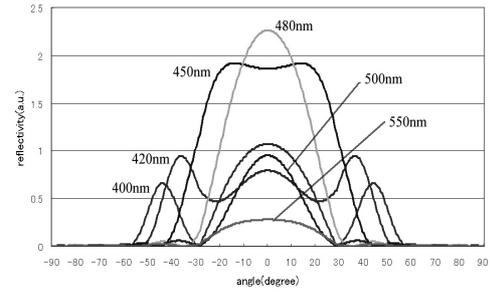


Fig. 14 the simulation result of angular dependence of reflected light intensity.

### 4.3. Rendering results

(fig.15) shows rendered images (left column) and the real photographs (right column). The rendered images matches well with real images qualitatively, especially in two points, 1) angular dependence of the reflectivity and 2) anisotropy of the scale reflection. For the former point, the light of 480nm wavelength reflects with the strongest intense and the wing show cyan color, which roughly corresponds to the real iridescent characteristics of *Morpho* wings.



Fig. 15 Rendered images (Left) and the real images (Right) of the wings of *Morpho* butterfly. Conditions: vertical sunlight, viewing angle 10, 30, 50 and 60 degrees from above to below.

#### 4.4. Evaluation of the dependence of color on the thicknesses of air and cuticle layers

For the simulation above, we assumed that the thicknesses of air and cuticle are fixed. But there must be biological deviation on the surface of actual *Morpho* wing. (Fig.16) shows the simulation results of the dependence of color on the thickness of air layer ( $d_{air}$ ). For this result, the thicknesses of cuticle layers ( $d_{cuticle}$ ) are assessed as follows:

$$(d_{cuticle}) = 235nm - (d_{air}).$$

With different thicknesses, the simulation results of reflectance colors shows dramatic variation, from green, blue, cyan to purple.

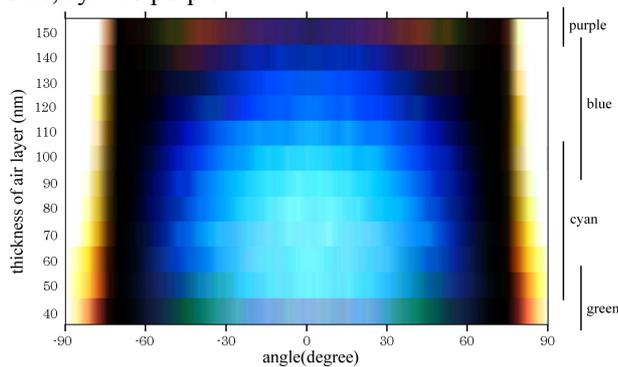


Fig. 16 dependence of color on the thicknesses of air and cuticle layers. ( $\omega = 90^\circ$ )

#### 5. Conclusion

We have developed a new reflectance model of structural color especially considering the micro structural anisotropy. And we presented shading methods for visualizing structural colors. Rendered images of *Morpho* butterfly by using our method matches well with real images qualitatively.

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