Tutorial ICAT 2004

Haptics for Immersive and Dynamic Virtual Worlds

Human interface Section, P&I Lab

Presenter

Makoto Sato

Email: msato@pi.titech.ac.jp

Shoichi Hasegawa

Email: hase@hi.pi.titech.ac.jp

Seungzoo Jeong

Email: jeongzoo@hi.pi.titech.ac.jp

Human Interface Section, Precision and Intelligence Laboratory, Tokyo Institute of Technology
Nagatsuda 4259, Midori-ku, Yokohama, Japan

http://sklab-www.pi.titech.ac.jp

Tutorial description

A introduction to virtual worlds with haptic sensation : its history, techniques, and recent advances, with a particular emphasis on string-based haptic interface SPIDAR. The first half of the course is a basic introduction to haptic devices and immersive virtual environment with haptic sensation. The second half covers several advanced techniques, including haptic rendering techniques, physically-based dynamic simulation for haptic interaction, and reactive virtual human . The real-time demonstrations using SPIDAR-system are programmed.

ICAT 2004

Contents

9:00-9:30 Introduction -- Sato

1. Overview of haptic interaction
2. String-based haptic device :SPIDAR

9:30-10:15 Immersive and Interactive virtual environments - Jeong

3. Immersive Virtual Environment (VE)
4. Interactive VE : Reactive Virtual Human

10:15-10:30 Break Time

10:30-11:15 Haptics in dynamic virtual worlds -- Hasegawa

5. Haptic interaction by SPIDAR
6. Real-time Rigid Body Simulation for Haptic Interactions

11:15-12:00 Demonstration and Discussion

7. Demo using SPIDAR system
8. Discussion

ICAT 2004

SPace Interface Device for Artificial Reality - SPIDAR -

2004.11.30 ICAT2004, Seoul

Precision & Intelligence Lab
Tokyo Institute of Technology
Makoto Sato

What is SPIDAR ?

"SPIDAR" is not "SPIDER"

What is SPIDAR ?

- Use strings as
  - Sensor

and

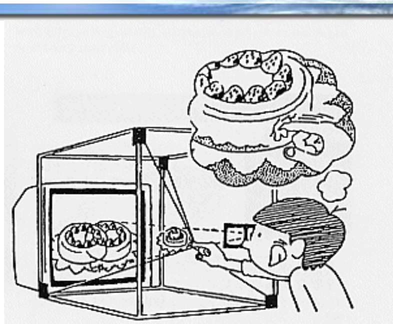
  - Actuator

OUTLINE

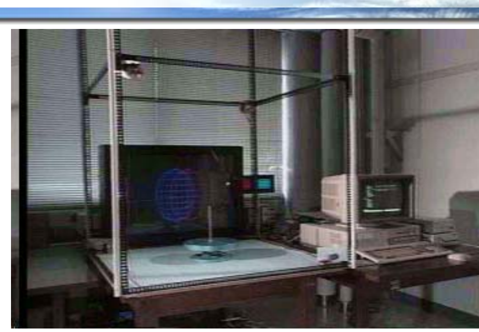
Virtual World of:

- Watch and Touch
- Pick and Place
- Peg in Hole
- Hand in Hand
- Open a Door
- Grasp and Move
- 4 + 4 Fingers

Watch and Touch



SPIDAR-I

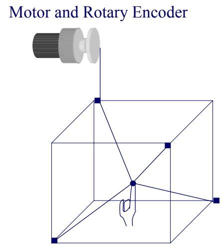


力覚ディスプレイ　SPIDAR

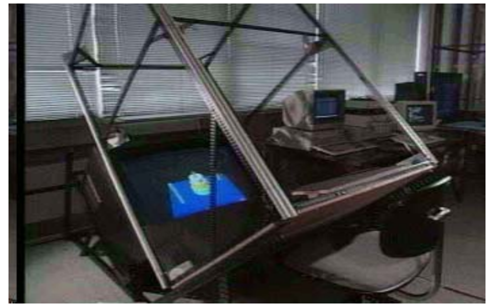
指先に対して4本の
糸が張られている

ロータリーエンコーダで
糸の長さを計測して
指先の3次元位置を計算する

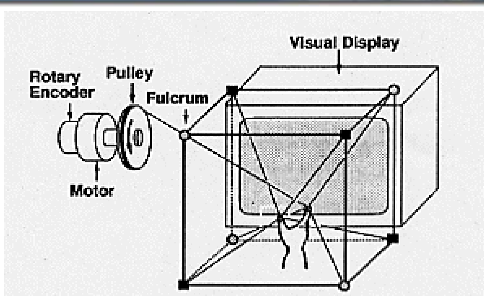
モータで糸の張力を制御して
指先に任意の力を加える



Pick and Place

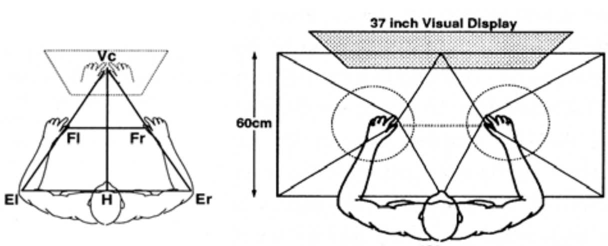


SPIDAR - II



Both hands manipulation

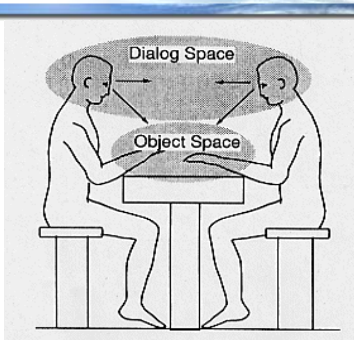
BOTH HANDS SPIDAR



Peg in Hole

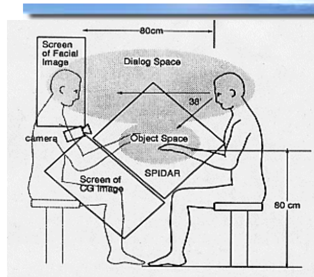
佐藤・小池研究室

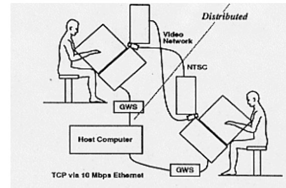
Hand in Hand



佐藤・小池研究室

NETWORKED SPIDAR



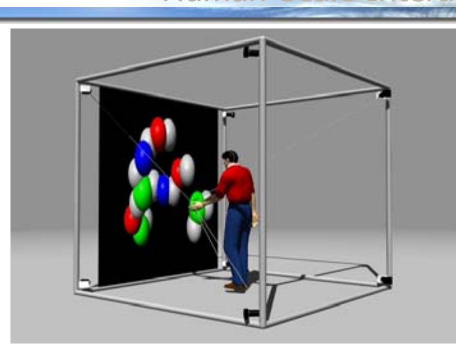


佐藤・小池研究室

NETWORKED SPIDAR

佐藤・小池研究室

Human-Scale Interaction



佐藤・小池研究室

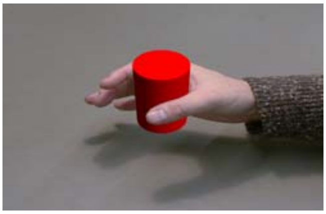
SPIDAR-H

佐藤・小池研究室

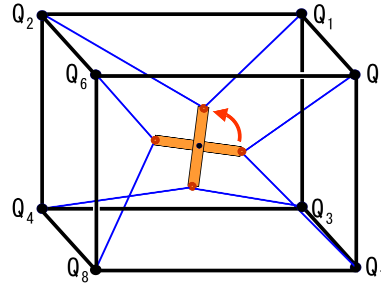
SIGGRAPH97

Grasp and Move

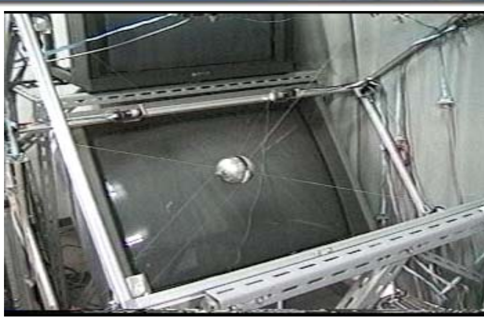
- ☑ Grasp object
- ☑ (6 + 1) DOF manipulation



SPIDAR-G



SPIDAR-G



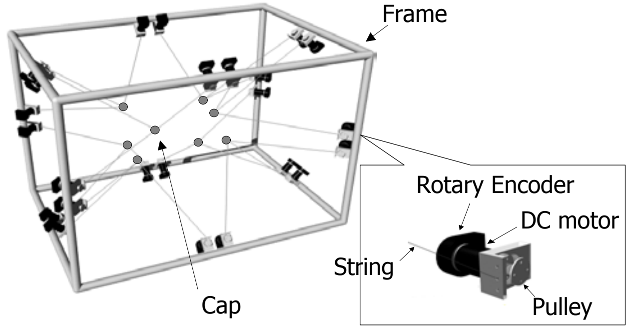
三次元グリップ

トレンドたまご

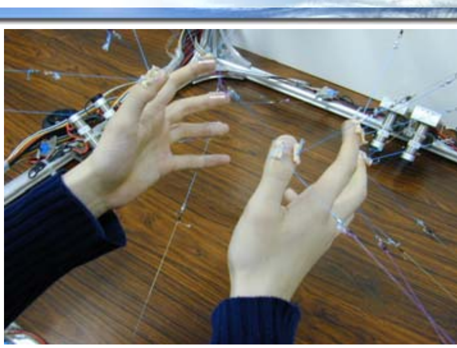
Both-handed SPIDAR-G

Multi-fingers Task

力覚提示装置：SPIDAR-8



SPIDAR-8



Virtual Rubik Cube

SIGGRAPH2000

Summary

	No of Strings
❑ SPIDAR I	4
❑ SPIDAR Ⅱ	8
❑ BOTH HANDS SPIDAR	16
❑ NETWORKED SPIDAR	16
❑ BIG SPIDAR	8
❑ SPIDAR-G	8
❑ SPIDAR-8	24

Summary

Features of SPIDAR

- Tension-based
- Finger-based systems
- Ground-referenced Force Feedback
- > 3DOF
- General Purpose

佐藤・小池研究室

Summary

Why string ?

- simple
- smooth
- safe

Immersive and Interactive Virtual Environment

Seungzoo Jeong
Makoto Sato's group
Precision and Intelligence Lab.
Tokyo Institute of Technology

ICAT 2004

Terminology

- 'Virtual Reality' refers to "Immersive Virtual Reality"
  - 'Artificial Reality'(1970s), 'Cyberspace'(1984), 'Virtual World' and 'Virtual Environment'(1990s)
  - " Virtual Reality is the use of computer technology to create the effect of an interactive three-dimensional world in which the objects have a sense of spatial presence."
- Immersion: "the extent to which computer displays are capable of delivering an inclusive, extensive, surrounding, and vivid illusion of reality to the senses of the VE participant." (Slater and Wilbur,'97)

ICAT 2004

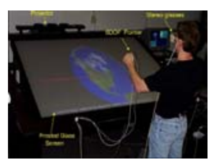
Presence?

- **Presence** is defined as "being there," and those are involvement and immersion
- **Social presence:** feeling that one is present with another person at a remote location
- **Virtual presence:** feeling as if present in a remote environment
- Factors which affect immersion include isolation from the physical environment, perception of self-inclusion in the virtual environment, natural modes of interaction and control, and perception of self-movement. (Witmer and Singer 1998,presence)

ICAT 2004

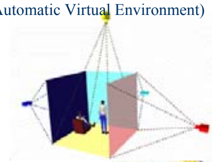
Types of VR System

- Non-immersive (desktop)
- Semi-immersive: Embedded without personal presentation equipment (ImmersaDesk, CAVE™,a table-size stereo display with head tracking)
- Fully immersive: Embedded inside the environment (CAVE, CYBERSPHERE ..)


Virtual workbench(1995-98)


ImmersaDesk2(1999)

CAVE(Cave Automatic Virtual Environment)



ICAT 2004

Performance of VR systems

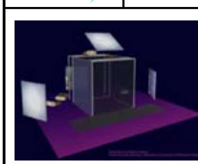
- Qualitative performance of different VR systems (Kalawsky, 1996)

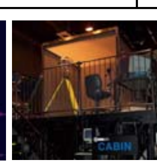
	Qualitative Performance		
Main Features	**Non- Immersive VR (Desktop)**	**Semi-Immersive VR (Projection)**	**Full Immersive VR (Head-coupled)**
Resolution	High	High	Low - Medium
Scale (perception)	Low	Medium - High	High
Navigation skills	Low	Medium	High
Field of regard	Low	Medium	High
Lag	Low	Low	Medium - High
Sense of immersion	None - low	Medium - High	Medium - High

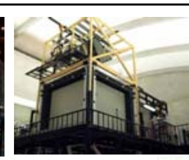
ICAT 2004

Immersive Projection System(1)

	Advantage	Disadvantage
CAVE, CABIN, COSMOS (Plane Screen)	Simple shape of screen to project High Performance is possible	Some distortion problems at the orthogonal joint area by screens







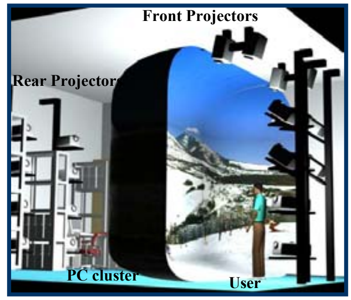
ICAT 2004

Immersive Projection System(2)

	Advantage	Disadvantage
Spherical Display (Curved Screen)	Continued joint area that can show the clear image without notification of that part	Hard to focus the image on the curved screen because of the characteristic of its shape


http://www.vr-systems.ndtilda.co.uk/sphere1.htm

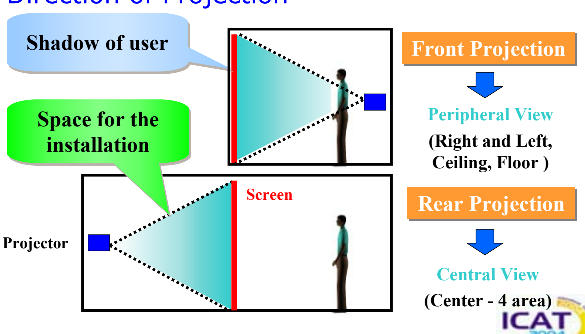
D-vision (Multi-Projection System)



- Duplex Vision : central view + peripheral view (2 FOV)
- Division : divided to 16 areas
- 4,500 x 3,500 pixel images by 24 PCs and 24 Projectors
- 180 degrees of view angle
- Screen size : 6.3m x 4.0m x 1.5m
- Stereoscopic image by linear polarized light

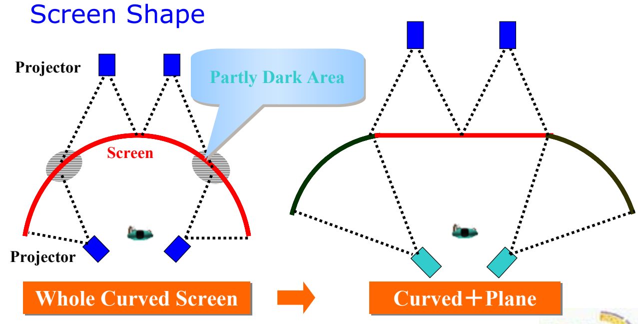
Feature of D-vision (1)

Direction of Projection



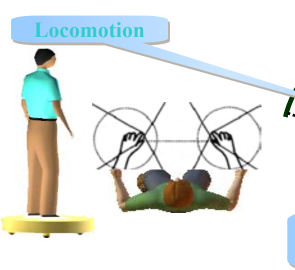
Feature of D-vision (2)

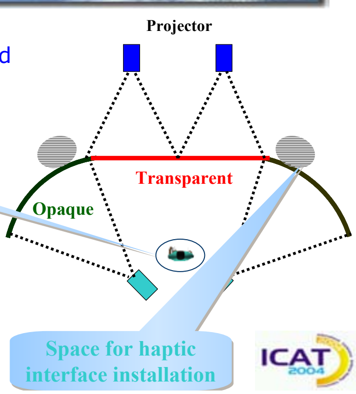
Screen Shape

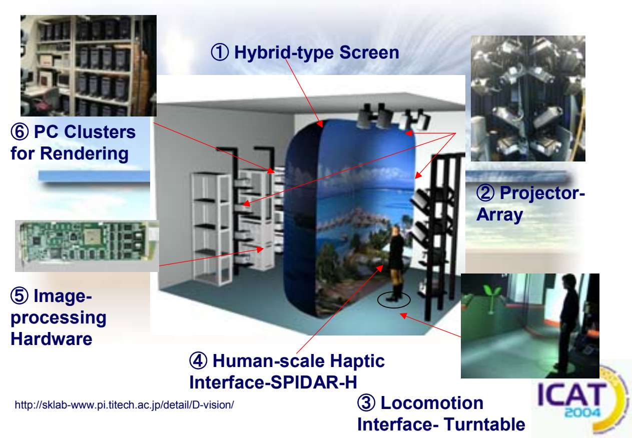


Feature of D-vision (3)

Flexible Integration:
Human-scale Haptic and Locomotion Interface, Cameras

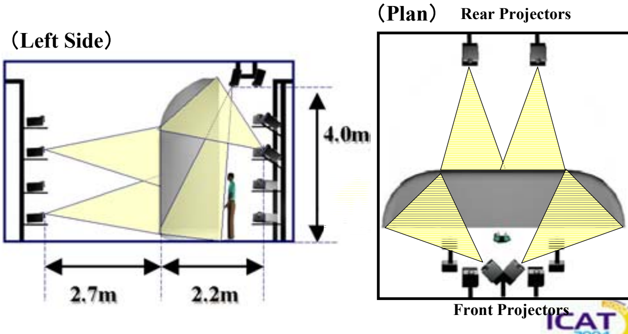




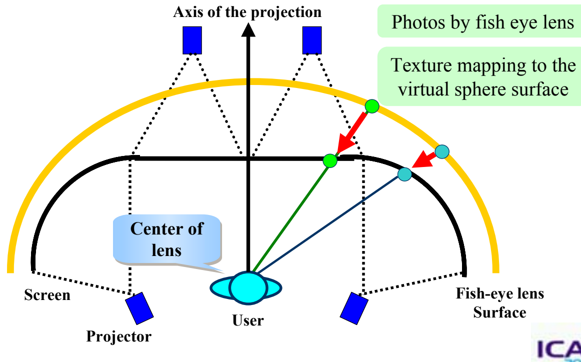


http://sklab-www.pi.titech.ac.jp/detail/D-vision/

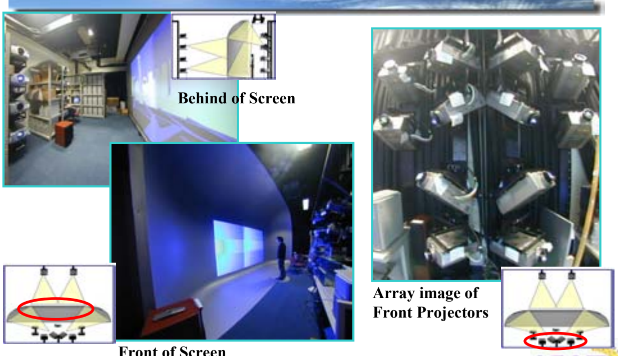
The position of Projectors



Projection method



Projector Installation



Distributed Rendering with PC Cluster

Computer hardware of PC cluster

New Spec.

- OS : WindowsXP or Linux
- CPU : Dual Pentium Ⅲ 800Mhz : (Pentium Ⅳ 2Ghz)
- Memory : 512 MB
- Graphic Card : NVIDIA GeForce 2 Ultra : (GeForce 4 Ti 4600)
- Network : 100Mbps Ethernet card
  \+ Myrinet [released by Myricom]
  full-duplex 2+2 Gbit/s data rate

Image Processing Hardware

Multi-Projection for Different Shaped
Hard to render image real-timely

Geometric Correction

Edge Blending

- Real time processing possible
- Geometric warping and color modulation of individual pixels

Model PA99

Model PA21

Image-Based Rendering

Real Photo Image

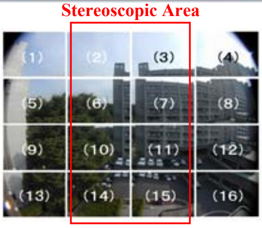
To experience virtual space

High-resolution
High-immersive

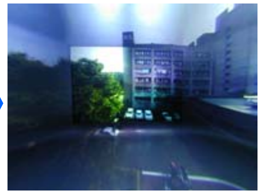
Easiest way to make virtual space contents

Generation of Seamless Images

Stereoscopic Area



16 Images for 24 PCs



Projected Images on Screen

24 PCs → Geometry Correction | Edge Blending → Real-time Image processor → Projector

Seamless Image Generation Method

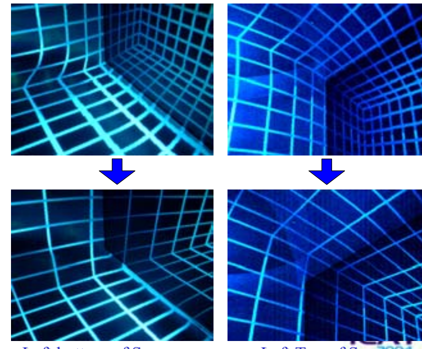
Existing･･･

Seamless Method from assumed one view

↓

Proposal･･･

Seamless Method for corresponding any view

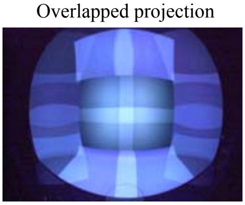


Left bottom of Screen

Left Top of Screen

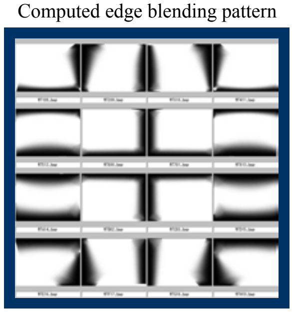
Edge Blending

Overlapped projection

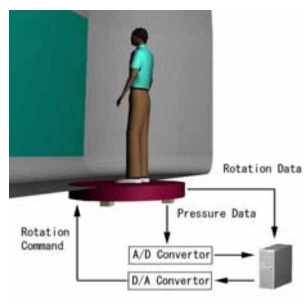


The relative positional relationship of pixels

Computed edge blending pattern



Locomotion Interface - Turntable



Organization of Turn-Table

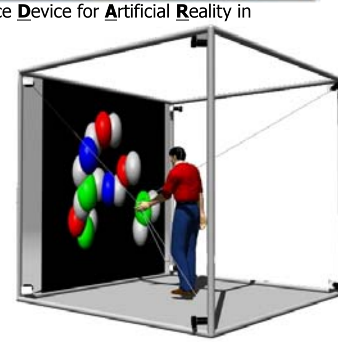
- Explore the virtual space freely with his natural stepping on the device
- Always face the front of the Screen
- No need to wear any gear

Turntable Demo

Human-scale Haptic Interface

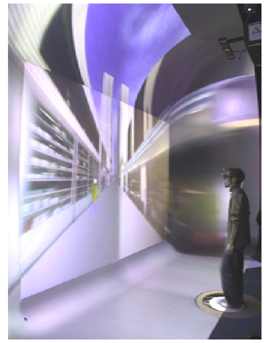
- SPIDAR-H (**SP**ace **I**nterface **D**evice for **A**rtificial **R**eality in Human Scale)
  - Human-scale
  - Wire-driven
  - Freedom of Movement
  - Smooth & Safe

Accuracy	ε ≤ 1.2cm
Max Force	Max.30N



SPIDAR-H DEMO

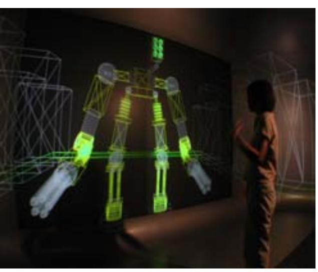
Applications



Urban Design Evaluation & Psychological Human Factors analysis (2001-2003)

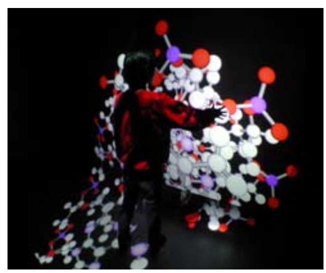
Applications

Design plan & Architectural Design



Applications

Education & Entertainment

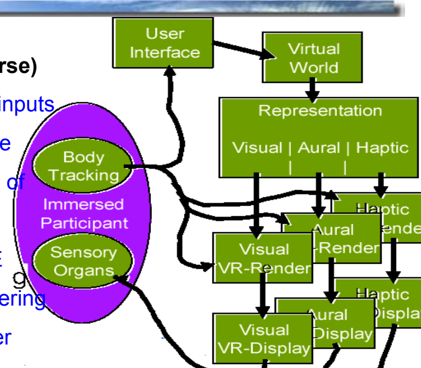


Dynamic and Interactive VE and Reactive Virtual Human

User Interfaces

The VR Process (siggraph 04, Course)

- Begins with user inputs to the User Interface
- Alter the internals of the virtual world
- Represent the VE
- User-centric rendering
- Display to the user



Interactive VE(1) – Control

Animating Athletic Motion Planning By Example (Graphics Interface ,*2000*)

Sympathetic interfaces:Using a plush toy to direct synthetic characters (CHI ,*1999*)

Using Motor, Sensors..

Mouse, Joystick, HMD..

Students Engaged in Virtual 'Field' Work (*SIGGRAPH 2003*)

Anyone for tennis? (wearing on mocap)

(presence ,*1999*)

Interactive VE(2)- Vision+Audio

Humanoid Agent: Gesture and Narrative language Recognition (*MIT ,2001*)

With the Virtual Football Trainer, you can move to any position on the field to experience the game from the player's perspective (Univ. of Michigan,2003)

Sensory System

- Audio Only
  - not good enough for full interaction
- Vision & Audio
  - comfort level but still ambiguous when interacting about specifics
- Virtual Reality environments allow people to communicate through multi-modal pathways
  - Social-presence
    - allows higher degree of interaction with others
  - Direct & Intuitive operation is possible

Dynamic & Intuitive Interaction?

- Accompany *Reactive motion* in the interaction that involves taking action in our daily life

  Ex) a handshake, hug, dance, sports..

  - *Interaction with Force Feedback* is an important communication!
  - We address it "responsive motion" which occurred by the force input from outside

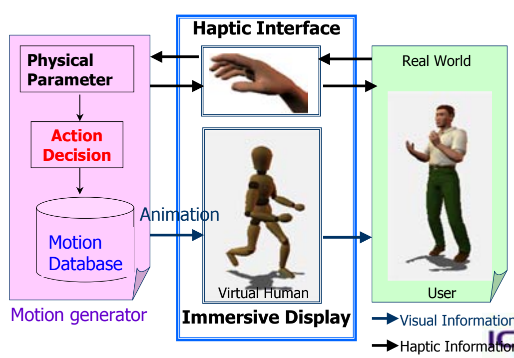
Reactive Virtual Human

Realize Reactive Virtual human which is capable of *Force interaction* with user

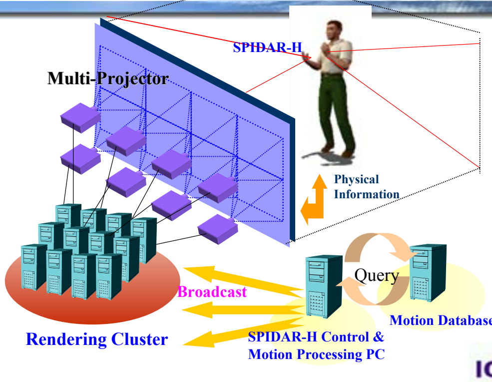
- *Behavioral realism*
- *Better active communication*
- *Intuitive and direct interaction with user*

It will be New Potential in other interaction system and human factor analysis, training task, entertainment applications..etc

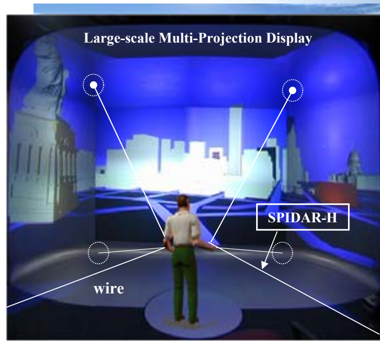
Basic Concept of Reactive VH



System Organization



Implemented System

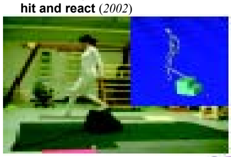


Motion generation

- Motion of Virtual Human
  - Dynamic simulation
  - Motion Capture
  - Combining motion capture and simulation

Motion Capture-driven simulation that hit and react *(2002)*

Locomotion simulation: Compassable Controller for Physics-based Modeling *(UCLA 2001)*



Interactive Control of Avatars Animated with Human Motion Data (Siggragh. *2002)*

Reactive Motion Generation

- Adopted *motion capture system* for constructing database of real movements
- *Reactive Motion* generation from *motion database and Haptic interface*
  - *Real-time* motion by physical parameter
  - *Rich* expression by motion data
  - *Best-fit* motion according to user's action

Reactive Virtual Human

"Virtual Catch Ball"

Future Works

- Problems to be solved
  - *Reinforce a grasping sensibility(interface)*
  - *Smoother motion generation of virtual human*
  - *Require better immediacy and intuitiveness of integrated system*
- *Challenges*
  - *Integrate other senses(tactile, gaze, hearing..)*
  - *Adopt combination method of Database and Kinematics*

Summary

- Described a multi-modal interaction system & applications in an immersive VE
- Introduced "Reactive Virtual Human"
  - *Realized <u>force feedback with user</u> in human-scale virtual environment*
  - *Generated <u>Reactive Motion</u> based on haptic Information from abundant motion data*

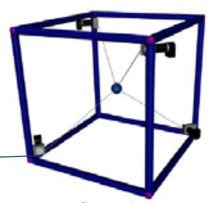
Haptic interaction by SPIDAR

Shoichi Hasegawa
Makoto Sato's group
Precision and Intelligence Lab.
Tokyo Institute of Technology

Haptic interaction with rigid bodies

- Control of SPIDAR
  - Characteristics of SPIDAR
  - Position measuring, Force displaying
  - Update rate for haptic rendering
- Rigid body simulation
  - Contact force modeling
  - Haptic rendering for 6DOF
  - Simulation of articulated body

Hardware of SPIDAR



Motor and encoder
Present force to user.
Measure length of string.

SPIDAR

Hardware performance

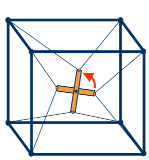
- SPIDAR is the best device in performance
  - Stiff and light

	SPIDAR	PHANTOM
Shape		
Stiffness	20N/mm	1N/mm
Weight	50g	75g

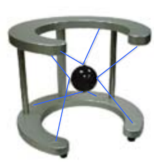
Reconfigurable hardware

- Any DOF and arrangements are designable.

3DOF
4Strings



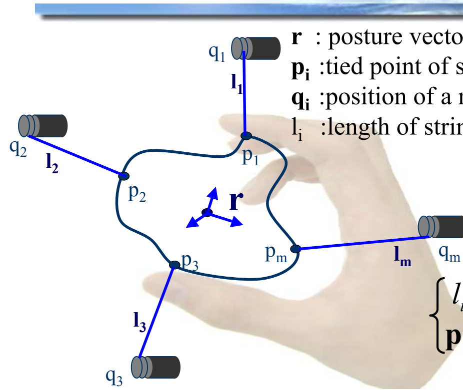
7DOF
8Strings



6DOF
8Strings

- Same control algorithm can be used

Position measurement



r : posture vector $(x, y, z, \theta_x, \theta_y, \theta_z)$
$\mathbf{p_i}$:tied point of string on the grip
$\mathbf{q_i}$:position of a motor
l_i :length of string

$$\begin{cases} l_i^2 = \|\mathbf{p_i} - \mathbf{q_i}\|^2 \\ \mathbf{p}_i = \mathbf{g}_i(\mathbf{r}) \end{cases}$$

Position measurement

$$\begin{cases} l_i^2 = \left\|\mathbf{p_i}-\mathbf{q_i}\right\|^2 \\ \mathbf{p}_i = \mathbf{g}_i(\mathbf{r}) \end{cases} \Rightarrow \begin{cases} \dfrac{\partial l_j}{\partial p_{ix}} = 2(p_{ix}-q_{ix}) \\ \dfrac{\partial p_{ix}}{\partial r_j} = \dfrac{\partial g'_{ix}}{\partial r_j} \end{cases}$$

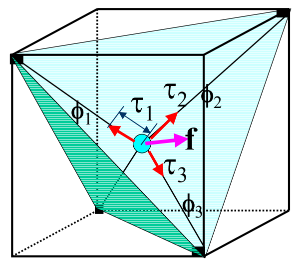
$$\Delta \mathbf{l} = \underbrace{\begin{pmatrix} \frac{\partial l_1}{\partial p_{ix}} & \frac{\partial l_1}{\partial p_{iy}} & \frac{\partial l_1}{\partial p_{iz}} \\ \vdots & \vdots & \vdots \\ \frac{\partial l_m}{\partial p_{ix}} & \frac{\partial l_m}{\partial p_{iy}} & \frac{\partial l_m}{\partial p_{iz}} \end{pmatrix}}_{\mathbf{J_p}^{\#}} \Delta \mathbf{p_i}, \quad \Delta \mathbf{p_i} = \underbrace{\begin{pmatrix} \frac{\partial p_{ix}}{\partial r_1} & \frac{\partial p_{iy}}{\partial r_1} & \frac{\partial p_{iz}}{\partial r_1} \\ \vdots & \vdots & \vdots \\ \frac{\partial p_{ix}}{\partial r_m} & \frac{\partial p_{iy}}{\partial r_m} & \frac{\partial p_{iz}}{\partial r_m} \end{pmatrix}}_{\mathbf{J_r}^{\#}} \Delta \mathbf{r}$$

$\Delta \mathbf{l} = \mathbf{J_p}^{\#} \mathbf{J_g}^{\#} \Delta \mathbf{r}$ ➡ Solve **r** by iterative method

Human interface Section, P&I Lab, Titech

Displaying force

❑ Simple solution



$$\mathbf{f} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_3 \end{pmatrix} (\tau_1 \cdots \tau_3)$$

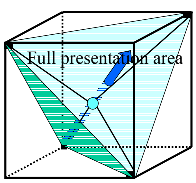
tensions

Directions of strings

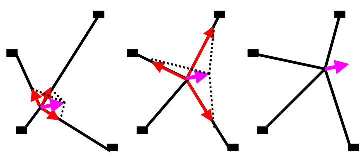
Human interface Section, P&I Lab, Titech

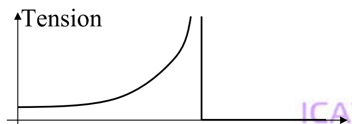
Displaying force

❑ Discontinuous problem



Partial presentation area





Human interface Section, P&I Lab, Titech

Displaying force

❑ Simple solution $\mathbf{f} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_3 \end{pmatrix} (\tau_1 \cdots \tau_3) \Leftrightarrow \sum_{i=1}^{4} \tau_i \phi_i - \mathbf{f} \to 0$

❑ Use smaller tension $\sum_{i=1}^{4} \tau_i^2 \to 0$

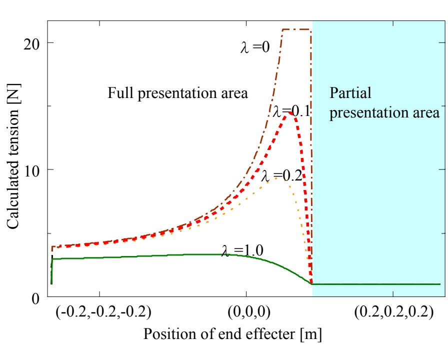
❑ Limitation $\tau_{\min} \le \tau_i, \tau_i \le \tau_{\max}$

❑ Finally $\sum_{i=1}^{4} \tau_i \phi_i - \mathbf{f} + \lambda \sum_{i=1}^{4} \tau_i^2 \to 0 \quad (\tau_{\min} \le \tau_i, \tau_i \le \tau_{\max})$

Quadratic programming problem

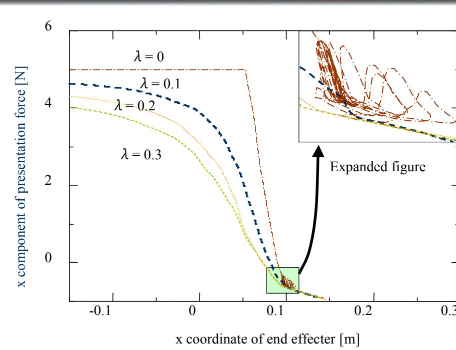
Human interface Section, P&I Lab, Titech

Displaying force

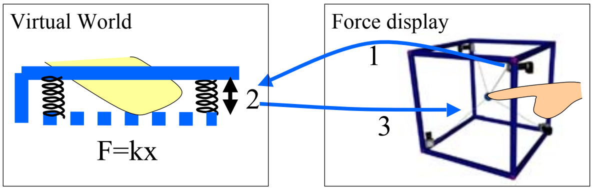


Human interface Section, P&I Lab, Titech

Displaying force



Haptic rendering and update rate

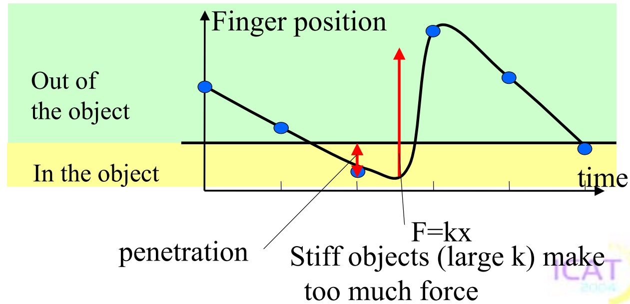


1. Measure finger position
2. Collision detection and force calculation
3. Display the force

Human interface Section, P&I Lab, Titech

Haptic rendering and update rate

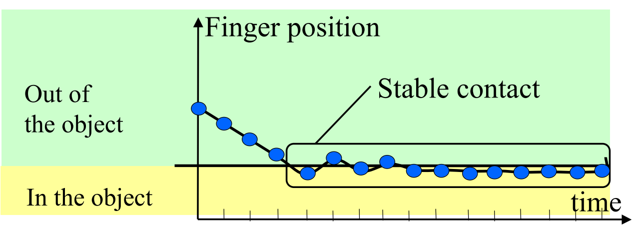
- Problem on slow update rate



Human interface Section, P&I Lab, Titech

Haptic rendering and update rate

- Solution by fast update rate

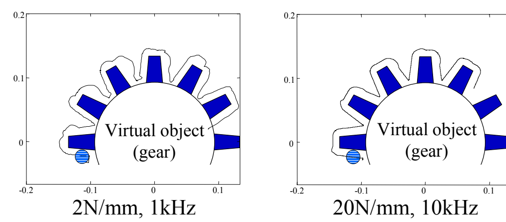


  - Stiff object requires fast update.
  - It is commonly said that 1kHz or more is needed.

Human interface Section, P&I Lab, Titech

Effect of fast update

- Advantage of stiffness
  - Display of friction disturbs display of shapes.
    But, enough stiffness realizes both.



Trajectory of the haptic pointer on surfaces with friction (μ=0.5)

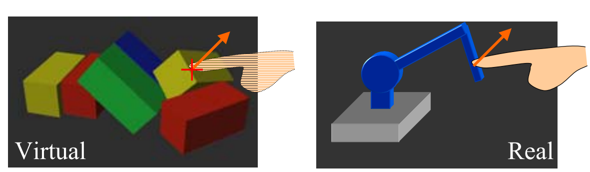
Human interface Section, P&I Lab, Titech

Real-time Rigid Body Simulation for Haptic Interactions

Human interface Section, P&I Lab, Titech

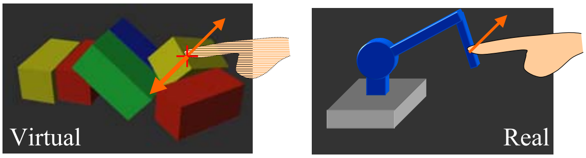
Haptic interaction

- Touch the virtual world
  - User feels contact force from haptic interface

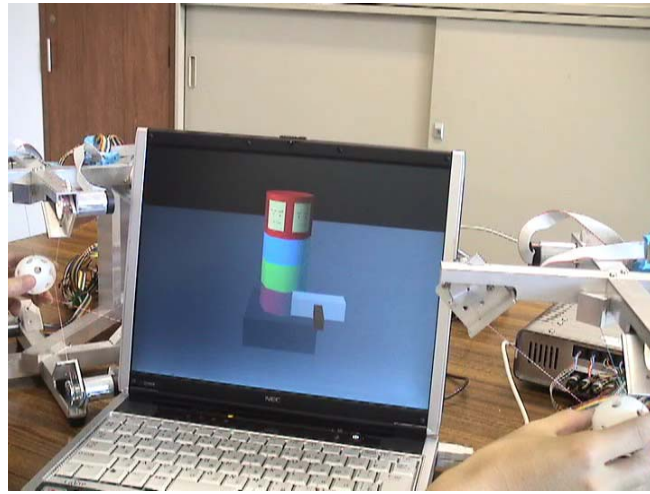


Haptic interaction

- Touch the virtual world
  - User feels contact force from haptic interface

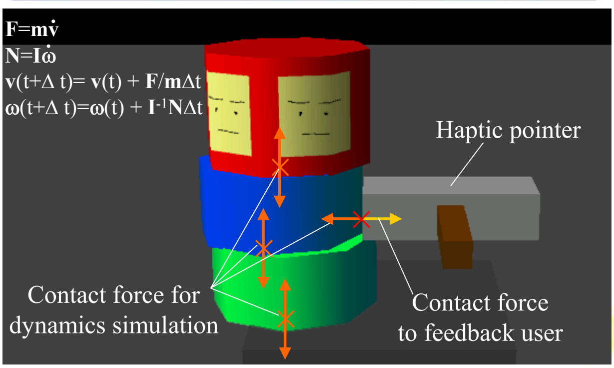


  - The touched object receives force from the user.
  - The response : Dynamics



Human interface Section, P&I Lab, Titech

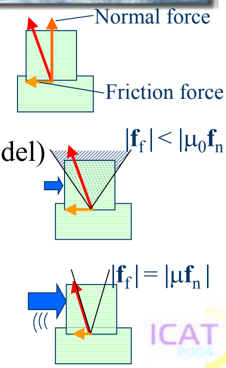
Contact force



Human interface Section, P&I Lab, Titech

Contact model

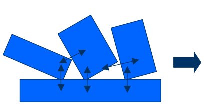
- Normal force
  - Prevent penetration
- Friction force (coulomb's model)
  - Static friction
    - Prevent sliding motion
  - Dynamic friction
    - Proportional to normal force



Human interface Section, P&I Lab, Titech

Solving constraints(1)

- Analytical method
  - David Baraff SIGGRAPH `89 …



$$\begin{cases} \mathbf{M}\ddot{\mathbf{r}}_i = \mathbf{f}_i & \text{(eq. of motion)} \\ (\ddot{\mathbf{p}}_i - \ddot{\mathbf{p}}_j)\cdot \mathbf{n}_{ij} \geq 0 & \text{(normal)} \\ \mathbf{f}\cdot\mathbf{n}/|\mathbf{f}| \geq \mu/\sqrt{1+\mu^2} & \text{(friction)} \end{cases}$$

Advantages

- Object motions are stable. Wide time steps are affordable.
- Solves constraints accurately. Completely rigid.

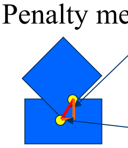
Drawbacks

- Much computation time for one step. $O(n^3)$
- A virtual coupling is needed to connect a haptic interface.
- Coulomb's friction model comes to NP complete problem.

Human interface Section, P&I Lab, Titech

Solving constraints(2)

- Penalty method



penetration $\mathbf{d}$, penetrating velocity $\dot{\mathbf{d}}$

$\mathbf{f}_n = k_n \mathbf{d} + b_n \dot{\mathbf{d}}$

Spring Damper

slide $\mathbf{l}$, sliding velocity $\dot{\mathbf{l}}$

$\mathbf{f}_f = k_f \mathbf{l} + b_f \dot{\mathbf{l}}$

Spring Damper

Advantages

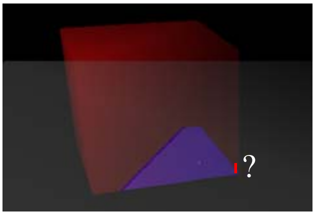
- Very fast for one step. O(n)
- Direct connection to haptic interfaces.
- Coulomb's friction model is easily realized.
- Integration of other models are easy. (e.g. Featherstone's method)

Drawbacks

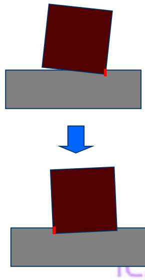
- Stability and rigidity requires small time steps. (Haptic interfaces also need this.)
- **Treatment of large contact area makes instability or takes a lot of computation time.**

Problem on large contact area

- Where should we put spring-damper model?

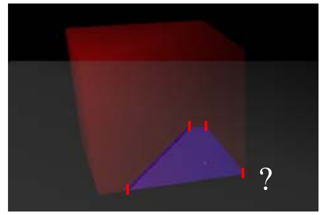


On the most penetrating point

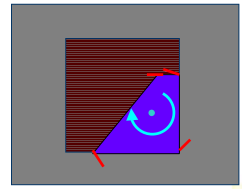


Problem on large contact area

- Where should we put spring-damper model?



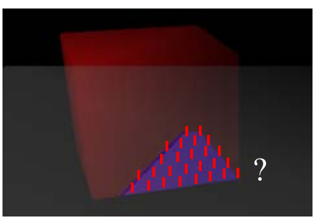
On vertices



Top view

Problem on large contact area

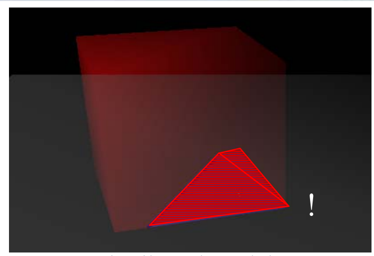
- Where should we put spring-damper model?



Many points

Will works well.
But, it will takes much computation time and memory.

Proposal for the problem



Distributed model

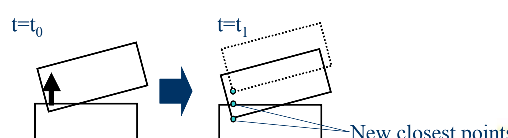
- Integrate forces from distributed model for each triangle.

Steps

- Finding Contact force:
  1. Find contact point and normal.
  2. Find the shape of the contact volume.
  3. Integrate forces over the contact area.

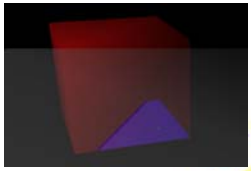
Contact detection

- Gilbert, Johnson, and Keerthi (GJK) algorithm.
  - Find closest points of two convex shapes.
    - A complex shape can be represented by a set of convex shapes.
    - After the contact, GJK can't find closest points, So…



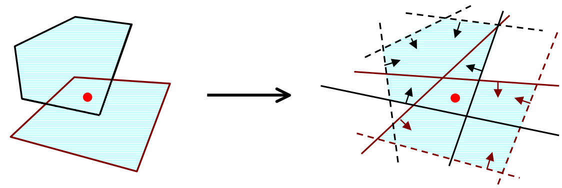
Contact Analysis

- Contact part = Intersection of two convexes.
- D. E. Muller and F.P.Preparata: "Finding the intersection of two convex" (1978)
  - For given two convex and a point in the intersection.
  - Find the intersection.



Contact Analysis(2)

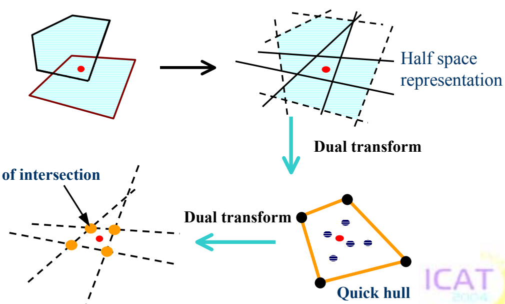
- Finding the intersection of two convex



Half space representation

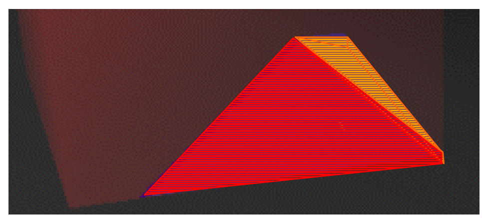
Contact Analysis(3)

- Finding the intersection of two convex(2)



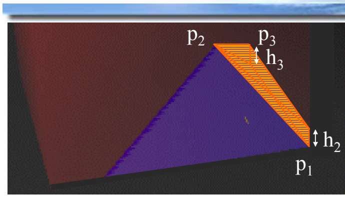
Integration of force

- Penalty force
- Dynamic friction force
- Maximum static friction force



Integrate forces from distributed model for each triangle.

Integration for a triangle



Force from spring model:

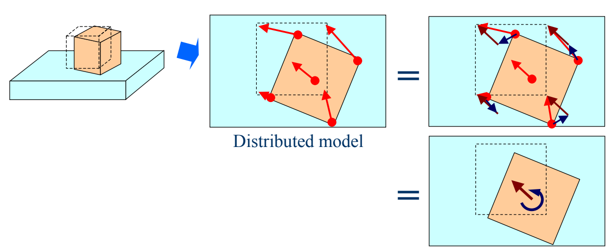
$$\mathbf{f} = k \int_{\mathbf{p} \in tri} h_p \mathbf{n} dS = k \frac{1}{3}(h_1 + h_2 + h_3)\ \mathbf{n}$$

Torque from from spring model:

$$\boldsymbol{\tau} = k \int_{\mathbf{p} \in tri} \mathbf{p} \times h_p \mathbf{n} dS = k \frac{1}{36}((h_1+h_2+h_3)\ (\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)+3(h_1\mathbf{p}_1+h_2\mathbf{p}_2+h_3\mathbf{p}_3))$$

Static friction force

- Spring-damper model for sliding constraint.



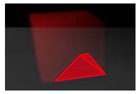
Distributed model

Two (translation and rotation) models

$$\mathbf{l} = \mathbf{r} + \boldsymbol{\theta} \times \mathbf{p}$$

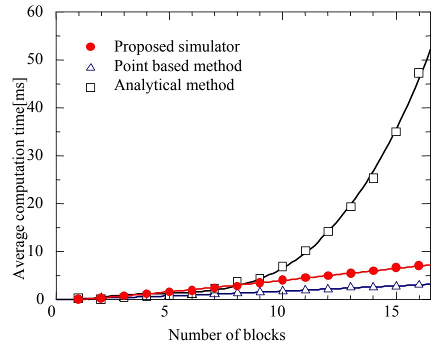
Evaluation

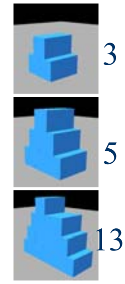
- Compare three simulators
  - Proposed
    - Penalty method
    - distributed model.
  - Point based
    - Penalty method
    - A model on the most penetrating point.
  - Analytic
    - Analytical method
    - Open Dynamics Engine (Smith R. 2000)



$$\begin{cases} \mathbf{M}\ddot{\mathbf{r}}_i = \mathbf{f}_i \\ (\ddot{\mathbf{p}}_i - \ddot{\mathbf{p}}_j)\cdot \mathbf{n}_{ij} \geq 0 \\ \mathbf{f}\cdot\mathbf{n}/|\mathbf{f}| \geq \mu/\sqrt{1+\mu^2} \end{cases}$$

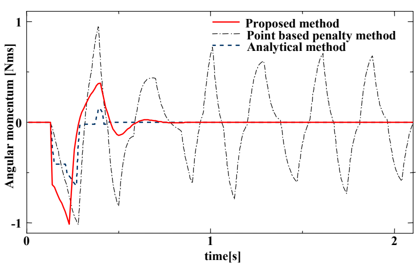
Computation time

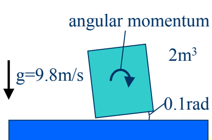




Stability on normal force

- A cube on a floor.
- Measure angular momentum.

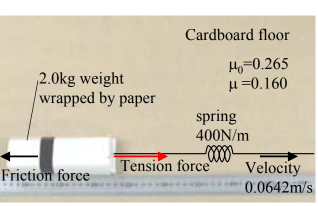


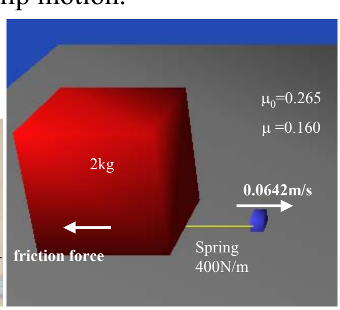


Motions of spinning tops with and without distributed friction

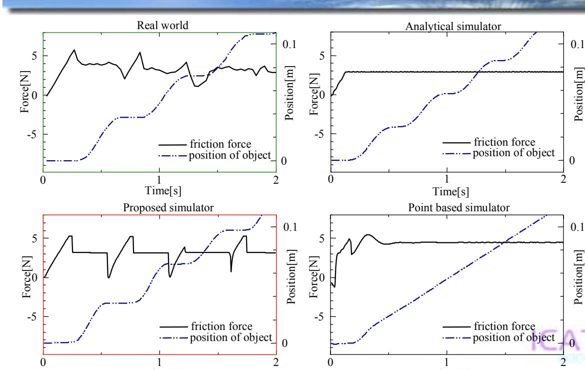
Stick-slip motion

- State transition between static and dynamic friction makes stick-slip motion.

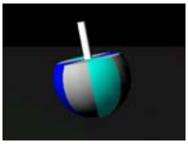
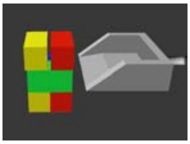
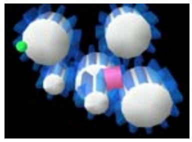
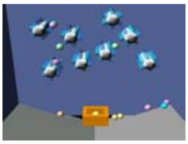




Result



Demo


Conclusion

- Proposed a real-time rigid body simulator for haptic interaction
  - Penalty method
  - Fast update rate
- Pointed out a problem on a large contact area
  - Solved the problem by integrating penalty over the intersection area
  - Fast and accurate simulation was achieved.

Thank you for listening

- Source codes, demos, movies...

http://springhead.info

Dual Transform

- Dual Transformation
  - Dual transformation transform a face into a vertex and a vertex into a face.
  - Dual transformation's dual transformation is original facet.