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BRDF Estimation System for Structural Colors

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Abstract

In this paper, we provide a shading algorithm for Structural Colors which can appear on micro surfaces. To approach a common algorithm for as many micro surfaces as possible, we modeled them out of polygons which have relative index of refraction. The algorithm calculates a surface's BRDF for each wevelength by ray tracing considering interference. Interference yield different BRDF between each wavelength, and we perceive it as structural colors. We simulated not only single thin film but also multilayered thin films as typical micro surfaces causing structural colors. We illustrated calculated BRDF in 3D computer graphics using environment mapping. Rendering results in our method indicate that it represent structural colors qualitatively.

Key words: Computer Graphics, Shading, Structural Colors, BRDF

1. Introduction

Colors which aren't generated by pigments but by interference or diffraction of light are called *Structural Colors*[4]. Examples of structural colors are showed on several natural objects : soap bubbles, morpho butterfly, jewel beetle and so on. Generally, structural colors are generated on micro surfaces making different BRDF for each wavelength of light. In this paper, we propose the method of creating BRDF of micro surfaces using ray tracing algorithm to generate structural colors. We produce BRDF not by analytic equation but by numerical simulation. In this method, we calculate interference as operations between complex numbers representing coherent lights.

It is no secret that microstructure of an object's surface determine that's reflection property. Ashikhmin.M et al.[1] modeled microstructures as sets of micro surfaces on 2D plain to calculate BRDF by shadowing between micro surfaces. Ashikhmin could represent anisotropic reflection on CG, but couldn't structural colors. Jay S. Gondek et al. [2] modeled surfaces as sets of particles which have pigment and index of refraction and calculated BRDF for each wavelength by using Monte Carlo ray tracing considering wavelength and polarization. Gondek represented not only that different distribution of pigment generate different reflection but also thin film interference on UV glass. Diffraction Shader[3] defined microstructure as height field function and could represent structural colors on CD and so on.

Our method is new BRDF estimation technique for structural colors shading, which can consider multilayer reflection. We explain how BRDF is estimated in chapter 2, how we implement the system in chapter 3. Then we cite some examples of calculated BRDF in chapter 4.

2. Exposition

2.1. BRDF

BRDF (Bidirectional Reflectance Distribution Function), which describes the reflection property of a surface, is defined as ratio of luminance to illuminance at a differential area[5]. Definition of Position-Invariant BRDF ρ is expressed as

$$\rho(\Theta_i, \Theta_r, \lambda) = \frac{dL_r(\Theta_i, \Theta_r, \lambda)}{dE_i(\Theta_i, \lambda)} = d\left(\frac{d\phi_r(\Theta_i, \Theta_r, \lambda)}{d\phi_i(\Theta_i, \lambda)}\right) \Big/ d\Omega_r \cos\theta_r ,$$

where E_i is the illuminance of incident light, L_r is the luminance of reflection light (subscript i and r express incident and reflection), ϕ is the light flux, Ω is the solid angle, λ is the wavelength. Θ is a spherical coordinate, which is expressed as

$$\Theta \in S^2 = \{(x, y, z) \in R^3 \mid x^2 + y^2 + z^2 = 1\} = \{(\theta, \varphi) \mid 0 \le \theta < \pi, 0 \le \varphi < 2\pi\} .$$

In case that lights reflect for only mirror reflect direction such as thin film and multilayered thin films, the reflect direction Θ_r of the ray is determined according to incident direction Θ_i as

$$\rho(\Theta_i, \Theta_r, \lambda) = 0 \qquad when \qquad (\theta_r, \varphi_r) \neq (\theta_i, \varphi_i + \pi).$$

In the case, the dimension of Θ_r is eliminated and BRDF is described as $\rho(\Theta_i, \lambda)$.

2.2. Light Expression and Interference

To calculate interference of reflection lights, each light has phase data. A 1D light wave in wave optics $F(x,t)$ is expressed as

$$F(x,t) = Ae^{(kx+\omega t+\delta)j},$$

where j is imaginary unit, k is wave number, ω is angular frequency, δ is initial phase.

If a coherent incident light is divided into two reflect lights, $F_1(l_1, t)$ and $F_2(l_2, t)$, in case that they interfere, interference is calculated as

$$\begin{aligned} F_1(l_1,t) + F_2(l_2,t) &= A_1 e^{(kl_1+\omega t+\delta)j} + A_2 e^{(kl_2+\omega t+\delta)j} \\ &\neq A_1 + A_2 \end{aligned},$$

generally, the sum of n coherent lights is

$$\sum_{k=1}^{n} F_k(l_k, t) = \sum_{k=1}^{n} A_k e^{(kl_j+\omega t+\delta)j},$$

where l_k is light path of each reflect lights. We supposed that the light flux of the reflect light for a direction is proportional to absolute value of the sum of coherent lights for the direction. Using proportionality factor K, this is expressed as

$$\phi_r = K\left|\sum_{k=1}^{n} F_k(l_k, t)\right|,$$

$$\begin{aligned} \frac{d\phi_r(\Theta_i, \Theta_r, \lambda)}{d\phi_i(\Theta_i, \lambda)} &= \left|\sum_{k=1}^{n} \frac{dF_k(l_k, t)}{dF(0,t)}\right| = \left|\sum_{k=1}^{n} \frac{dA_k}{dA} e^{kl_k j}\right| \\ &= \left|\sum_{k=1}^{n} \frac{dA_k}{dA} e^{\frac{2\pi l_k}{\lambda} j}\right| \end{aligned}.$$

Reflection light fluxes $d\phi_r(\Theta_i, \Theta_r, \lambda)$ are estimated from incident light fluxes $d\phi_i(\Theta_i, \lambda)$ using ray tracing which calculate amplitude A_k and light path l_k for each reflect light waves.

In BRDF estimation, rays which have same direction of incident and reflect are coherent, and added as complex number (This means interference). In the computer, whether two lights have same direction is evaluated in digitized direction.

2.3. Luminance Estimation

In the rendering stage of CG, the luminance for a reflect direction $L_r(\Theta_r, \lambda)$ is estimated from given illuminance field $E_i(\Theta_i, \lambda)$ as below,

$$dL_r(\Theta_i, \Theta_r, \lambda) = \rho(\Theta_i, \Theta_r, \lambda) dE_i(\Theta_i, \lambda)$$

$$\begin{aligned} L_r(\Theta_r, \lambda) &= \oint_{\Theta_i} dL_r(\Theta_i, \Theta_r, \lambda) \\ &= \int_{\Theta_i} \rho(\Theta_i, \Theta_r, \lambda) dE_i(\Theta_i, \lambda) \end{aligned}$$

In the case that the dimension of Θ_r is eliminated, luminance estimation is expressed as

$$\begin{aligned} L_r(\Theta_r, \lambda) &= \rho(\Theta_i, \lambda) E_i(\Theta_i, \lambda) \\ (\theta_r, \varphi_r) &= (\theta_i, \varphi_i + \pi) \end{aligned}.$$

2.4. RGB color system

Transformation from wavelength-oriented BRDF to BRDF represented as RGB color system is calculated as below

$$X(\Theta_i, \Theta_r) = K\int_{380}^{780} \rho(\Theta_i, \Theta_r, \lambda)\bar{x}(\lambda) d\lambda$$

$$Y(\Theta_i, \Theta_r) = K\int_{380}^{780} \rho(\Theta_i, \Theta_r, \lambda)\bar{y}(\lambda) d\lambda$$

$$Z(\Theta_i, \Theta_r) = K\int_{380}^{780} \rho(\Theta_i, \Theta_r, \lambda)\bar{z}(\lambda) d\lambda$$

$$K = 1 \Big/ \int_{380}^{780} \bar{y}(\lambda) d\lambda$$

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.5064 & -1.7400 & -0.5441 \\ -1.0690 & 1.9777 & 0.0352 \\ 0.0563 & -0.1970 & 1.0511 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix},$$

where $\bar{x}, \bar{y}, \bar{z}$ is color-matching function of XYZ color system[8].

2.5. Ray Tracing

Our ray tracing method is for BRDF estimation, whose target objects are sets of polygon representing micro surfaces. Polygon model can simulate multilayer reflection. These polygons are nanometer-scale so they can cause interference. Only adding phase data to rays, we can't consider diffraction and polarization. Differences between our ray tracing and classic one are as follows.

1) Rays reflect followed by Fresnel reflection.

2) Rays are described as complex numbers for calculation interference.

3) Ray tracing is executed for each wavelength.

Polygons are positioned in the surface coordinate system whose space is spanned by normal vector **n**, tangent vector **t**, binormal vector **b**, which is a kind of local coordinate system. Ray tracing is also executed in the surface coordinate system (Fig.1). Fig.2 shows diagrammic illustration of BRDF estimation.

Our ray tracing algorithm works as below,

1) It creates a ray as incident light and initializes the ray data. The intensity of the ray is 1, the direction is Θ_i, the position is same with light source's, the light path is 0, current refraction index is 1. The ray is added in "rays list".

2) If one or more rays exist in rays list, it takes a ray from the list. It names this ray R. If no ray exist, ray tracing is terminated.

3) If a polygon P which collides with R, it calculates the position of collision x. it calculates light path L from the position and current refraction index of R, and x. If P doesn't exist, go to 7).

4) It calculates the reflect ray and the refract ray of R on x. The reflection ratio and the transmission ratio are calculated by using Fresnel's formulas and multiplied by each ray.

5) The position of the reflect ray and the refract ray of R become x. Their light paths is R's plus L. The refract ray's current refraction index is index of R multiplied by relative refraction index of P.

6) It adds both rays into rays list. go to 2).

7) If n (normal vector) component of the direction of R is positive, R is added into "reflect rays list". Complex number expression of R is determined by intensity, light path, wavelength of R.

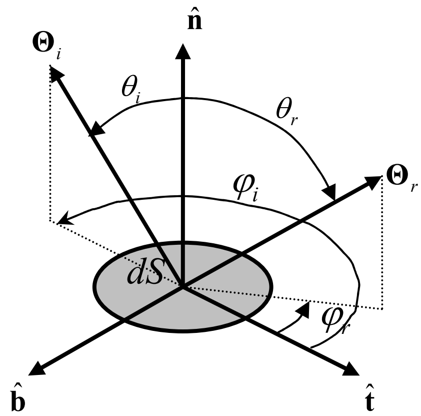


Figure 1: Surface coordinate system

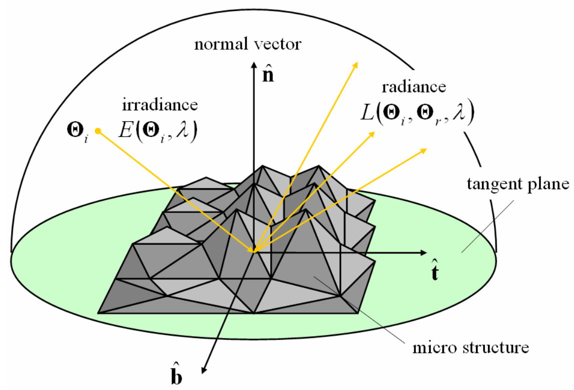


Figure 2: BRDF estimation image

3. Implementaion

3.1. BRDF Estimation System

To estimate BRDF of the micro surface, the system has 4 stages of work as below,

1) We define micro surface using polygon.

2) The system executes ray tracing algorithm to simulate incident and reflect rays for the micro surface.

3) It obtains wavelength oriented BRDF table by calculating interference for each wavelength.

4) It obtains BRDF table represented as RGB color system by using color-matching function.

3.2. BRDF Table

In the computer, BRDF must be digitized[6][7]. BRDF ρ for a wavelength in continuous system is expressed as a following map,

$$\rho : S^2 \times S^2 \rightarrow R \, .$$

As described in section 2.2, S^2, which represent directions, is digitized (sampled). Thus BRDF is 4D table in the computer. BRDF table represented as RGB color system can be regard as a 4D texture on the rendered surface. In a similar reason, elimination of Θ_r dimension brings a 2D texture.

3.3. Lighting

In the rendering stage, the illuminance represented as RGB $E_i(\Theta_i)$ is given by the environment map for simplicity. The environment map $Env(\Theta)$ is defined in world coordination. Using the function L, which maps from surface coordinate to world coordinate, the illuminance is calculated as

$$E_i(\Theta_i) = Env(L(\Theta_i)),$$

L is can be described as below

$$L = \begin{pmatrix} \mathbf{t}^T \\ \mathbf{n}^T \\ \mathbf{b}^T \end{pmatrix}^{-1},$$

in other words, L is matrix multiplication representing liner transformation of world basis.

4. Simulation and Result

4.1. Thin Film

Thin film is the most popular microstructure causing structural colors. Fig. 3 shows definition of thin film and a rendered image of the teapot which has calculated BRDF on its surface by Microsoft DirectX. As it shows, thin film can be defined by polygons representing two plains. In Fig. 3, refraction index of the thin film is 2.4 and thickness is 300nm. The lower base is regarded as a perfect specular surface here. In the image of BRDF, center of image is origin, at which there is a color we look at perpendicularly, radial direction indicates θ_i part, argument direction φ_i part.

In the 3D image, the teapot is illuminated by white light from all direction. We can see the structural colors of red-violet on its surface.

4.2. Multilayered Thin Films

Multilayered thin films can be seen on some natural lives such as insects. Having more degrees of freedom of parameters, multilayered thin films can cause more complex structural colors than single thin film. Fig. 4 shows multilayered thin films teapot. The layer in Fig. 4 has 3 films of which refraction index is 2.4. It has 180nm in thickness at each film and each space between films.

The BRDF say we can see color of orange, as we look at the surface perpendicularly. As view line is more horizontal, we can see green and blue.

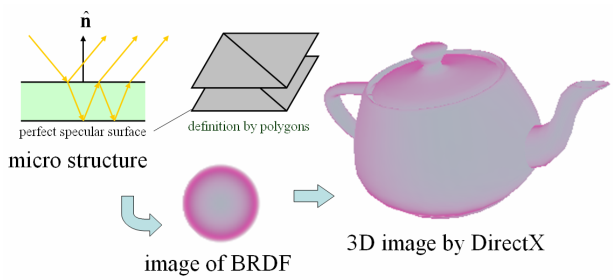


Figure 3: BRDF of a thin film

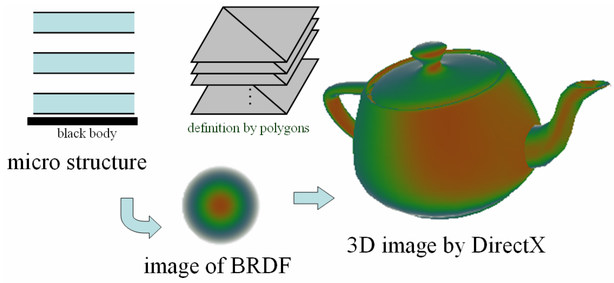


Figure 4: BRDF of a multilayered thin films

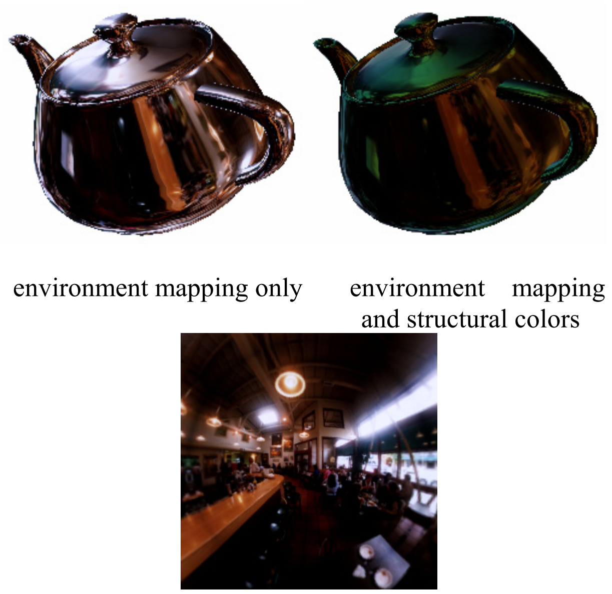


Figure 5: Structural colors image using environment mapping

4.3. Environment map

Fig. 5 shows the rendered using environment mapping on the right, and the teapot added structural colors effect showed in Fig. 4 on the left. In comparison to being illuminated by white light from all direction (Fig. 4), teapot is seen more naturally by using environment mapping.

4.4. Sampling of Wavelength

Fig. 6 shows that different microstructures bring different necessary sample number of wavelength. About the example in Fig. 6, thin film whose thickness is 300nm needs at least 10 samples of wavelength. But in case that thickness is 3000nm, it needs 40 samples (When less than that, we see strange colors which should not be in real BRDF). This result indicates that the more long light path rays go, the more samples of wavelength we need.

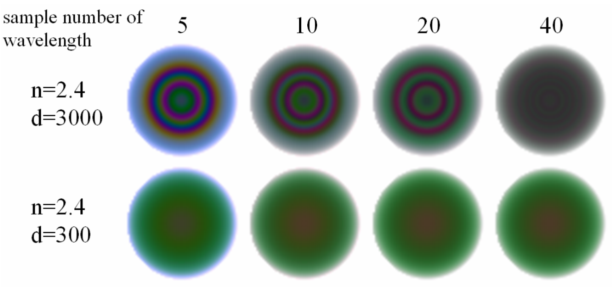


Figure 6: Sampling of wavelength

4.5. Comparison with formula for analysis

Fig. 7 shows the results of formula for analysis and ray tracing for the same microstructure. Target microstructure is single thin film. For simplicity, sampled wavelengths are 436nm(blue), 546nm(green), 700nm(red) only. Both calculations consider interference and Fresnel reflection. The result indicates ray tracing simulation matches well with formula for analysis qualitatively.

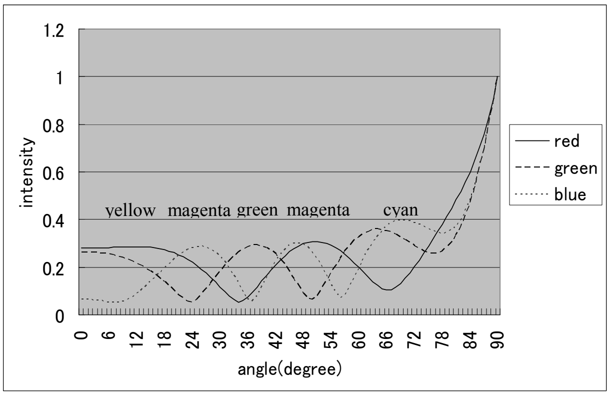
5. Conclusion

We have been successful in simple examples. Rendering results in our method represent structural colors from thin film and multilayered thin films qualitatively as follos,

1) Colors generated on the surfaces which illuminated only by which light without pigments.

2) Colors change observed as the view point is changed.

We are trying for bumpy micro surfaces by using distributed ray tracing.



Result of formula for analysis

Result of ray tracing

Figure 7: Comparison of ray tracing with formula for analysis

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